

**V úlohách 1.3.5 – 1.3.8 určte definičný obor daných funkcií.**

**1.3.5** a)  $f_1: y = \frac{\sqrt{x+1}}{x-4}$       d)  $f_4: y = \frac{\sqrt{2x+10}}{16-x^2}$

b)  $f_2: y = \sqrt{\frac{-3}{x^2-5x+4}}$       e)  $f_5: y = \frac{1}{\sqrt{x}} + \sqrt{x^2-5}$

c)  $f_3: y = \frac{-3}{\sqrt{x^2-3x}}$       f)  $f_6: y = \frac{\sqrt{15+2x-x^2}}{8-2x}$

**1.3.6** a)  $f_1: y = 4^{\log(2x^2-5x-3)}$       d)  $f_4: y = \log x^2 + \log(4-x^2)$

b)  $f_2: y = \ln \sqrt{\frac{3x-1}{x+4}}$       e)  $f_5: y = \sqrt{1-\log(x^2+7x+10)}$

c)  $f_3: y = \frac{-1}{\ln(2x-x^2)}$       f)  $f_6: y = \frac{\sqrt{\ln(x-1)}}{x-2}$

**1.3.7** a)  $f_1: y = \arcsin \frac{2x+4}{x}$       d)  $f_4: y = \frac{1}{x} + \arccos(x^2-1)$

b)  $f_2: y = \operatorname{arccotg} \frac{x^2}{x^2-2}$       e)  $f_5: y = \frac{x}{\operatorname{arctg}(12-4x)}$

c)  $f_3: y = \arccos \frac{1}{x^2}$       f)  $f_6: y = \sqrt{\arcsin(x-4)}$

**1.3.8** a)  $f_1: y = \log(1-2x) - 3 \arcsin \frac{3x-1}{2}$

b)  $f_2: y = 5 \log \left( \frac{x+1}{x-5} \right) - \frac{\sqrt{5x-10}}{x^2-36}$

c)  $f_3: y = \arccos(3+2x) + \sqrt{\frac{x-2}{x+3}}$

d)  $f_4: y = \frac{\sqrt{x^2-5x+6}}{\ln(2x-5)} - \sqrt{5-x}$

e)  $f_5: y = \frac{\sqrt{x^2-x-2}}{\ln x} - 4 \cdot \arcsin \frac{1-2x}{4}$

f)  $f_6: y = \sqrt{\log \frac{5x-x^2}{4}} + \frac{1}{\log_2 x-2}$

g)  $f_7: y = \arccos \frac{3}{2x-5} + \ln(6+11x-2x^2)$

h)  $f_8: y = \frac{14-x}{\ln(x^2-4)} + \arccos(3x+7)$

**V úlohách 2.2.1 a 2.2.4 vypočítajte limitu postupnosti:**

**2.2.1** a)  $\lim_{n \rightarrow \infty} (-2n^5 + 3n^2 - 10)$

h)  $\lim_{n \rightarrow \infty} \left( \frac{5n - n^5}{2n^5 + n} \right)^4$

b)  $\lim_{n \rightarrow \infty} \left( 3n^4 - \frac{3}{n^3} + 4n \right)$

i)  $\lim_{n \rightarrow \infty} \frac{(n+3)(2n-1)}{(n-3)^2}$

c)  $\lim_{n \rightarrow \infty} \frac{3-8n}{n+2}$

j)  $\lim_{n \rightarrow \infty} \sqrt{\frac{9+n^2}{4n^2}}$

d)  $\lim_{n \rightarrow \infty} \frac{4n^2}{-n+2}$

k)  $\lim_{n \rightarrow \infty} \frac{3n+2}{\sqrt{16n^4 + 6n^3 - 2}}$

e)  $\lim_{n \rightarrow \infty} \frac{7n+2n^3}{7n^2 - n - 1}$

f)  $\lim_{n \rightarrow \infty} \frac{-3n^2 + 4n}{4n^3 - n + 2}$

g)  $\lim_{n \rightarrow \infty} \left( \frac{3n-5}{4+3n} \right)^5$

**2.2.4** a)  $\lim_{n \rightarrow \infty} \left( 1 + \frac{1}{5n} \right)^{2n+6}$

g)  $\lim_{n \rightarrow \infty} \left( \frac{6+4n}{2+4n} \right)^{3-2n}$

b)  $\lim_{n \rightarrow \infty} \left( 1 + \frac{7}{3n} \right)^{n-1}$

h)  $\lim_{n \rightarrow \infty} \left( \frac{7n+10}{1+7n} \right)^{\frac{n}{3}}$

c)  $\lim_{n \rightarrow \infty} \left( \frac{n+5}{n+4} \right)^{2n-1}$

i)  $\lim_{n \rightarrow \infty} \left( \frac{3n+6}{3n-1} \right)^n$

d)  $\lim_{n \rightarrow \infty} \left( \frac{2n+5}{2n} \right)^{3n-7}$

j)  $\lim_{n \rightarrow \infty} \left( \frac{2n-5}{2n-2} \right)^{4n^2}$

e)  $\lim_{n \rightarrow \infty} \left( \frac{2n+1}{2n-3} \right)^{3n}$

k)  $\lim_{n \rightarrow \infty} \left[ \ln \left( \frac{3n+1}{3n-5} \right)^{2-n} \right]$

f)  $\lim_{n \rightarrow \infty} \left( \frac{n-1}{n+3} \right)^{5-4n}$

l)  $\lim_{n \rightarrow \infty} (n+1) \cdot [\ln(n+1) - \ln(n+2)]$

**3.1.1** Vypočítajte limity funkcií:

$$\begin{aligned} \text{a)} \quad & \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 2x}, \\ \text{b)} \quad & \lim_{x \rightarrow 0} \frac{x^3 - 4x}{2x^2 + 3x}, \\ \text{c)} \quad & \lim_{x \rightarrow -1} \frac{x^3}{(x+1)^2}, \\ \text{d)} \quad & \lim_{x \rightarrow -1} \frac{(x+1)^2(x-1)}{x^3 + 1}, \end{aligned}$$

**3.1.2** Zistite, ktorá z daných funkcií je spojitá.

$$\begin{aligned} \text{a)} \quad f : y = & \begin{cases} \frac{x^2 - 9}{x+3}, & x \neq -3 \\ 6, & x = 3 \end{cases} \\ \text{b)} \quad f : y = & \begin{cases} \frac{x^3 - 4x}{x-2}, & x \neq 2 \\ 8, & x = 2 \end{cases} \\ \text{c)} \quad f : y = & \begin{cases} 3+x^2, & x \leq 0 \\ \frac{\sin 3x}{x}, & x > 0 \end{cases} \end{aligned}$$

**3.1.3** Určte parameter  $a$  tak, aby funkcia  $f$  bola spojité:

$$\begin{aligned} \text{a)} \quad f : y = & \begin{cases} ax, & x < 1 \\ 2 - \frac{x}{a}, & x \geq 1, \end{cases} & \text{b)} \quad f : y = & \begin{cases} e^{ax}, & x < 0 \\ a - x, & x \geq 0, \end{cases} \\ \text{c)} \quad f : y = & \begin{cases} \frac{-1}{x^2 + a}, & x < 0 \\ \left(\frac{1}{3}\right)^x, & x \geq 0 \end{cases} & \text{d)} \quad f : y = & \begin{cases} e^{-\frac{1}{x}}, & x \leq -1 \\ x^2 + ax, & x > -1 \end{cases} \end{aligned}$$

**3.1.4** Nájdite asymptoty grafu funkcie

$$\begin{aligned} \text{a)} \quad f : y = & \frac{x}{x+4} & \text{f)} \quad f : y = xe^{\frac{1}{x^2}} \\ \text{b)} \quad f : y = & \frac{1-x^2}{x-2} & \text{g)} \quad f : y = 4xe^{-x^2} \\ \text{c)} \quad f : y = & \frac{2x^2}{2x-1} & \text{h)} \quad f : y = x + e^{-x} \\ \text{d)} \quad f : y = & 3x + \frac{3}{x-2} & \text{i)} \quad f : y = x + \frac{\ln x}{x} \end{aligned}$$

**4.1. Napíšte rovnicu dotyčnice a normály ku grafu funkcie  $f$  v bode  $T$ .**

- |  |  |
|--|--|
| a) $f : y = x^2 - 2x + 2, T = [0, ?]$      | e) $f : y = \frac{2x}{x+1}, T = [0, ?]$                    |
| b) $f : y = 2x - x^2, T = [1, ?]$          | f) $f : y = x + \sqrt{4-x}, T = [3, ?]$                    |
| c) $f : y = 1 - \frac{1}{x+1}, T = [0, ?]$ | g) $f : y = (x-1) \cdot e^x, T = [1, ?]$                   |
| d) $f : y = \sqrt[3]{x+4}, T = [-3, ?]$    | h) $f : y = \ln \sin x, T = \left[\frac{\pi}{2}, ?\right]$ |

**4.2. Napíšte rovnicu dotyčnice ku grafu funkcie  $f$ , ktorá zviera s osou  $x$  uhol  $45^\circ$ .**

- |                                     |                                      |
|-------------------------------------|--------------------------------------|
| a) $f : y = 2 \cdot \sqrt{x^2 + 3}$ | b) $f : y = \operatorname{arctg} 2x$ |
|-------------------------------------|--------------------------------------|

**4.3. Napíšte rovnicu dotyčnice a normály ku grafu funkcie  $f$ , ak dotyčnica  $t$  je rovnobežná s danou priamkou  $p$ .**

- |   |   |
|---|---|
| a) $f : y = \ln(x+1), p : x - y + 2 = 0$                      | c) $f : y = x^3 - x, p : 2x - y = 0$          |
| b) $f : y = 3 - 2 \cdot e^{\frac{x}{2}}, p : 2x + 2y - 3 = 0$ | d) $f : y = \frac{2x-1}{2-x}, p : 3x - y = 0$ |

**4.4. Dané sú funkcie celkových nákladov a príjmov. Vypočítajte hodnoty marginálnych nákladov, príjmov a zisku pre danú úroveň produkcie  $x$  a výsledky ekonomicky interpretujte.**

- |   |
|---|
| a) $C(x) = 600 + 20x, R(x) = 30x, x = 50$                             |
| b) $C(x) = x^2 - 6x + 25, R(x) = 25x - 2x^2, x = 5$                   |
| c) $C(x) = 50 + 3x - 0,01x^2, R(x) = 2,5x - 0,005x^2, x = 25$         |
| d) $C(x) = \frac{x^3}{3} - 0,5x^2 + 40, R(x) = 150x - 0,2x^2, x = 10$ |
| e) $C(x) = 100 \cdot e^{0,01x}, R(x) = 10x \cdot e^{0,01x}, x = 100$  |

**4.5. Vypočítajte marginálny dopyt pre jednotkovú cenu  $p$ , ak je daná funkcia dopytu  $d$ . Výsledky ekonomicky interpretujte.**

- |                               |   |
|-------------------------------|---|
| a) $d : q = 56 - 2p, p = 6$   | c) $d : q = 1 + 3e^{-\frac{p}{3}}, p = 3$ |
| b) $d : q = 100 - p^2, p = 4$ | d) $d : q = \frac{60}{p+3} - 2, p = 2$    |

**4.6. Daná je dopytová funkcia  $d$ . Vypočítajte elasticitu dopytu pre jednotkovú cenu  $p$ . Výsledky ekonomicky interpretujte.**

- |  |                                      |
|--|--------------------------------------|
| a) $d : q = 27 - 3p, p = 3$            | c) $d : q = \frac{12}{p} - 3, p = 2$ |
| b) $d : q = \frac{32 - p^2}{2}, p = 4$ | d) $d : q = 80 - 30\sqrt{p}, p = 4$  |

**5.1. Nájdite intervaly monotónnosti funkcie  $f$ .**

- |   |  |
|---|--|
| a) $f : y = 2x^2 + x - 6$                       | h) $f : y = \frac{2}{x^2 + 1}$         |
| b) $f : y = x^3 - 27x$                          | i) $f : y = \frac{2x}{x^2 + 1}$        |
| c) $f : y = 4x^3 + 3x^2 - 6x + 2$               | j) $f : y = \frac{2}{1-x^2}$           |
| d) $f : y = 1 - 12x - 9x^2 - 2x^3$              | k) $f : y = \frac{x-1}{x^2}$           |
| e) $f : y = 3 - 2x^2 + 4x^4$                    | l) $f : y = \frac{2}{x} + \frac{x}{2}$ |
| f) $f : y = \frac{x^5}{5} - \frac{4}{3}x^3 + 1$ | m) $f : y = \frac{x^2}{2-x}$           |
| g) $f : y = \frac{x^6}{6} + \frac{x^5}{5}$      | n) $f : y = x + \frac{x}{x^2 - 1}$     |

**5.2. Nájdite intervaly monotónnosti funkcie  $f$ .**

- |                                       |   |
|---------------------------------------|---|
| a) $f : y = \sqrt{1-x}$               | h) $f : y = x \cdot \sqrt{4-x^2}$           |
| b) $f : y = \sqrt[3]{(x+2)^2}$        | i) $f : y = \frac{2}{3}x + \sqrt[3]{x^2}$   |
| c) $f : y = \sqrt{\frac{1+x}{1-x}}$   | j) $f : y = 2x - 3 \cdot \sqrt[3]{(1-x)^2}$ |
| d) $f : y = (x-3) \cdot \sqrt{x}$     | k) $f : y = x + e^{-x}$                     |
| e) $f : y = x \cdot \sqrt{1-x}$       | l) $f : y = 2^{x^2-6x+2}$                   |
| f) $f : y = x \cdot \sqrt{1+x^2}$     | m) $f : y = \frac{e^{2x-x^2}}{2}$           |
| g) $f : y = \frac{x-3}{\sqrt{1+x^2}}$ | n) $f : y = x \cdot e^x$                    |

**5.3. Nájdite intervaly monotónnosti funkcie  $f$ .**

- |   |  |
|---|--|
| a) $f : y = x \cdot e^{\frac{1}{x}}$    | h) $f : y = \ln(x^2 - 4)$                    |
| b) $f : y = x \cdot e^{-\frac{x^2}{2}}$ | i) $f : y = x \cdot \ln x$                   |
| c) $f : y = (1+x^2) \cdot e^{-x^2}$     | j) $f : y = \frac{\ln x}{x}$                 |
| d) $f : y = \frac{e^x}{x+1}$            | k) $f : y = \frac{\ln(3-x)}{3-x}$            |
| e) $f : y = x - 2 \ln x$                | l) $f : y = \ln\left(\frac{x+2}{2-x}\right)$ |
| f) $f : y = 2x^2 - \ln x$               | m) $f : y = \frac{2}{x} + \ln x^2$           |
| g) $f : y = \ln(1-x)$                   | n) $f : y = \frac{x}{\ln x}$                 |

**6.1. Nájdite intervaly konvexnosti a konkávnosti funkcie  $f$ . Určte inflexné body funkcie, ak existujú.**

a)  $f : y = x^3 - 9x^2 + 1$

h)  $f : y = x + \frac{1}{x}$

b)  $f : y = x^4 - 2x^3 - 7$

i)  $f : y = 3x + \frac{1}{2x^2}$

c)  $f : y = x^4 + 4x^3 - 18x^2 + 3x + 2$

j)  $f : y = \frac{3x^2}{1-x}$

d)  $f : y = x^4 - x^5$

k)  $f : y = \frac{x^2 + x + 21}{x + 2}$

e)  $f : y = \frac{x^6}{6} - \frac{x^5}{4} + 3$

l)  $f : y = \frac{2x}{1+x^2}$

f)  $f : y = 3x - (4-x)^5$

m)  $f : y = \frac{x}{1-x^2}$

g)  $f : y = x^4 + 2x^3 + 6x^2$

n)  $f : y = \frac{x^2}{16-x^2}$

**6.2. Nájdite intervaly konvexnosti a konkávnosti funkcie  $f$ . Určte inflexné body funkcie, ak existujú.**

a)  $f : y = \frac{x^2 + 1}{x^2 - 1}$

e)  $f : y = 3x - \sqrt{x-3}$

b)  $f : y = \frac{1}{x^3} + \frac{1}{x^2}$

f)  $f : y = 4 + \sqrt[3]{x^2}$

c)  $f : y = \frac{1}{x^3} - \frac{6}{x}$

g)  $f : y = \frac{2x}{\sqrt{x^2 + 1}}$

d)  $f : y = \left( \frac{1}{2} + \frac{1}{x} \right)^2$

h)  $f : y = \frac{x}{\sqrt{x^3 + 1}}$

**6.3. Nájdite intervaly konvexnosti a konkávnosti funkcie  $f$ . Určte inflexné body funkcie, ak existujú.**

a)  $f : y = x \cdot e^{-x}$

f)  $f : y = e^{4-\frac{x^2}{2}}$

b)  $f : y = e^{-x^2}$

g)  $f : y = e^{1-\frac{x^3}{3}}$

c)  $f : y = x \cdot e^{-x^2}$

h)  $f : y = e^{2x} - 8e^x + 5x$

d)  $f : y = x^2 \cdot e^{-x}$

i)  $f : y = (2 - x^2) \cdot e^{-x}$

e)  $f : y = x \cdot e^{\frac{1}{x}}$

j)  $f : y = \frac{e^x}{x}$

**7.1. Použitím l'Hospitalovo pravidla vypočítajte limity .**

- |   |  |
|---|--|
| a) $\lim_{x \rightarrow 1} \frac{x^3 - 2x^2 + x}{x^2 - 1}$                    | h) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$   |
| b) $\lim_{x \rightarrow 3} \frac{x^3 - 9x}{x^4 - 3x^3 - x + 3}$               | i) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x^2}$  |
| c) $\lim_{x \rightarrow -1} \frac{x^4 + x^3 - 2x^2 - 3x - 1}{x^4 + 4x^2 - 5}$ | j) $\lim_{x \rightarrow \frac{\pi}{3}} \frac{1 - 2 \cos x}{\pi - 3x}$  |
| d) $\lim_{x \rightarrow 1} \frac{\sqrt[3]{8x} - 2x}{\sqrt[4]{x} - x}$         | k) $\lim_{x \rightarrow 2} \frac{3 \operatorname{tg} \pi x}{2 - x}$  |
| e) $\lim_{x \rightarrow 1} \frac{\sqrt{x+1} - \sqrt{2}}{x^2 - 1}$             | l) $\lim_{x \rightarrow 0} \frac{\sin x}{e^x - 1}$   |
| f) $\lim_{x \rightarrow 0} \frac{5^x - 1}{x}$                                 | m) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\operatorname{arctg} \left( x - \frac{\pi}{2} \right)}{\pi - 2x}$ |
| g) $\lim_{x \rightarrow 1} \frac{3x^3 - 3}{3^x - 3}$                          | n) $\lim_{x \rightarrow 0} \frac{\operatorname{arctg} 3x}{\arcsin 2x}$   |

**7.2. Použitím l'Hospitalovo pravidla vypočítajte limity .**

- |  |   |
|--|---|
| a) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{2x - \sin x}$                     | j) $\lim_{x \rightarrow -1} \frac{(x^2 + 3x + 2)^2}{x^3 - 3x - 2}$        |
| b) $\lim_{x \rightarrow 0} \frac{x^3 + \pi x}{\sin 3x}$                          | k) $\lim_{x \rightarrow 0} \frac{1 - \cos^4 x}{4x^2}$                     |
| c) $\lim_{x \rightarrow 0} \frac{\ln(1 + 4x)}{3^x - 1}$                          | l) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{2x \cdot \sin x}$            |
| d) $\lim_{x \rightarrow 0} \frac{\sin 4x}{\ln(1 + \sin x)}$                      | m) $\lim_{x \rightarrow \frac{\pi}{6}} \frac{\ln(\sin 3x)}{(6x - \pi)^2}$ |
| e) $\lim_{x \rightarrow 0} \frac{3 \ln(1 - 2x)}{2 \operatorname{arctg} 3x}$      | n) $\lim_{x \rightarrow 0} \frac{x - \sin x}{e^x - e^{-x} - 2x}$          |
| f) $\lim_{x \rightarrow 0} \frac{\operatorname{arctg} x + x^2}{2^{3x} - 3^{2x}}$ | o) $\lim_{x \rightarrow 0} \frac{e^{3x} - 3x - 1}{\sin^2 x}$              |
| g) $\lim_{x \rightarrow 0} \frac{e^{3x} - e^{-2x}}{2 \arcsin x - \sin x}$        | p) $\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{\sin^2 x}$                  |
| h) $\lim_{x \rightarrow 0} \frac{\ln(\cos 3x)}{\operatorname{arctg} 4x}$         | q) $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{e^{x^2} - 1}$               |
| i) $\lim_{x \rightarrow 0} \frac{(1+x)^2 - (1+2x)}{x^2 + 4x^3}$                  | r) $\lim_{x \rightarrow 0} \frac{x^3}{x - \operatorname{arctg} x}$        |

**8.1. Nájdite lokálne extrémy funkcie  $f$ , ak existujú.**

a)  $f : y = x^2 - 6x + 1$   
 b)  $f : y = 3 + 10x - 5x^2$   
 c)  $f : y = x^3 - 3x^2 - 9x + 7$

d)  $f : y = x - \frac{16}{3}x^3$   
 e)  $f : y = (x+2)^2(x+5)$   
 f)  $f : y = -(1-x)(x-3)^2$

g)  $f : y = -(x+1)^2(x-3)^2$   
 h)  $f : y = (2x+1)^2(2x-1)^2$   
 i)  $f : y = \frac{x^4}{4} - \frac{x^3}{3}$   
 j)  $f : y = x^4 + 2x^3 - 3$   
 k)  $f : y = 1 - \frac{x^5}{5} - \frac{x^4}{4}$   
 l)  $f : y = x^5 + x^3 + 1$

**8.2. Nájdite lokálne extrémy funkcie  $f$ , ak existujú.**

a)  $f : y = \frac{x}{2} + \frac{2}{x}$

f)  $f : y = 1 + \frac{1}{x^2 - x}$

b)  $f : y = \frac{x^2}{x+3}$

g)  $f : y = \frac{x^3}{x^2 + 1}$

c)  $f : y = \frac{2x+1}{x^2}$

h)  $f : y = \frac{x^4 + 1}{x^2}$

d)  $f : y = \frac{x}{1+x^2}$

i)  $f : y = \frac{1}{x^4 - 1}$

e)  $f : y = \frac{1+x^2}{1-x^2}$

j)  $f : y = \frac{(x+1)^2}{x^2 - 2x}$

**8.3. Nájdite lokálne extrémy funkcie  $f$ , ak existujú.**

a)  $f : y = x - \sqrt{x-1}$

e)  $f : y = 2 - \sqrt[3]{(2-x)^2}$

b)  $f : y = 2x + \sqrt{2x-1}$

f)  $f : y = 3 \cdot \sqrt[3]{(x+1)^2} - 2x$

c)  $f : y = 4 - \sqrt[3]{x}$

g)  $f : y = x \cdot \sqrt{9-x}$

d)  $f : y = 1 - \sqrt[3]{x^2}$

h)  $f : y = (5-x) \cdot \sqrt[3]{x^2}$

**8.4. Nájdite lokálne extrémy funkcie  $f$ , ak existujú.**

a)  $f : y = x \cdot e^{-x}$   
 b)  $f : y = (4-x)e^{4-x}$   
 c)  $f : y = (x^2 + 1) \cdot e^{-x}$   
 d)  $f : y = e^{-x^2}$

e)  $f : y = x \cdot e^{-\frac{x^2}{2}}$   
 f)  $f : y = x + e^{-x}$   
 g)  $f : y = e^x + e^{-x}$   
 h)  $f : y = \frac{e^x}{x+1}$

**9.1. Nájdite lokálne extrémy funkcie  $f$ , ak existujú.**

a)  $f : y = x \cdot \ln x$

f)  $f : y = \ln^2 x - 2 \ln x$

b)  $f : y = \frac{x^2}{2} - \ln x$

g)  $f : y = \ln\left(\frac{x-1}{x+1}\right)$

c)  $f : y = \frac{1}{x} + \ln x$

h)  $f : y = \frac{\ln x}{x^2}$

d)  $f : y = \frac{1 + \ln x}{x}$

i)  $f : y = \frac{\ln x}{\sqrt{2x}}$

e)  $f : y = \ln(4x - x^2)$

j)  $f : y = \frac{\ln^2 x}{x}$

**9.2. Celkové náklady na výrobu  $x$  jednotiek určitého tovaru sú dané funkciou  $C(x) = 0,01x^3 + 10x + 160$ . Aký veľký musí byť objem produkcie, aby priemerné náklady boli najmenšie.**

**9.3. Pre danú funkciu dopytu a funkcie celkových nákladov zistite množstvo výrobkov, ktoré sa musia vyrábať a predávať, aby sa dosiahol maximálny zisk. Určite hodnotu maximálneho zisku a predajnú cenu, ktorá tento zisk maximalizuje.**

a)  $p(x) = 18 - 2x$

c)  $p(x) = 30 - \frac{27}{2}x$

$C(x) = 3 + 6x$

$C(x) = 3 + 6x - x^3$

b)  $p(x) = 45 - 3x$

d)  $p(x) = 60 - 27x$

$C(x) = 5x + x^2$

$C(x) = 6 + 12x - 2x^3$

**9.4. Pre každú funkciu dopytu v nasledujúcich úlohách určte množstvo produkcie, pre ktoré sa dosahujú maximálne príjmy a predajnú cenu, ktorá je výsledkom týchto príjmov.**

a)  $p(x) = 21 - 0,7x$

c)  $p(x) = 6e^{-0,02x}$

b)  $p(x) = 24 - 0,5x^2$

d)  $p(x) = \sqrt{243 - 9x}$

**9.5. Je daná funkcia dopytu  $p(x) = 45 - 0,5x$  a funkcia priemerných nákladov**

$\tilde{C}(x) = x^2 - 8x + 57 + \frac{2}{x}$ . Určte množstvo

a) predanej produkcie v hl, ktoré maximalizujú príjmy;

b) vyrobenej produkcie v hl, ktoré minimalizujú marginálne náklady;

c) vyrobenej a predanej produkcie v hl, ktoré maximalizuje celkový zisk.

**V úlohách 3.1.1. až 3.1.5. vypočítajte neurčité integrály (na intervaloch, v ktorých existujú):**

**3.1.1.** a)  $\int (5x^2 - 4x + 10) dx$

c)  $\int (x^2 - 4x \cdot \sqrt[3]{x} + 10 \cdot \sqrt[4]{x^3}) dx$

e)  $\int (x^2 - 2)^3 dx$

g)  $\int \left( \frac{3}{x} - \frac{7}{x^3} + \frac{6}{\sqrt[3]{x}} - \frac{18}{x\sqrt{x}} \right) dx$

i)  $\int (1 - 3x + x^3) \cdot \sqrt[3]{x} dx$

b)  $\int x \cdot (3x - 4)^2 dx$

d)  $\int (x - 2)(4 - x) dx$

f)  $\int (x^2 - \frac{5}{2}x + 6) \cdot (x^3 + 1) dx$

h)  $\int x \cdot \sqrt{x \cdot \sqrt{x}} dx$

j)  $\int \sqrt{x \cdot \sqrt{x^3}} dx$

**3.1.2.** a)  $\int (e^x - e^3) dx$

c)  $\int \frac{15^x - 9^x}{3^x} dx$

b)  $\int 5^x \cdot e^x dx$

d)  $\int \frac{(3^x + 4^x)^2}{12^x} dx$

**3.1.3.** a)  $\int (\sin x - 3\cos x) dx$

c)  $\int \frac{1}{\cos^2 x \cdot \sin^2 x} dx$

b)  $\int (\cot^2 x) dx$

d)  $\int \frac{2}{x^2 + 9} dx$

**3.1.4.** a)  $\int \frac{\cos x}{10 + \sin x} dx$

c)  $\int \frac{1}{x \cdot \ln x} dx$

b)  $\int \frac{\sin x}{2 + 5 \cdot \cos x} dx$

d)  $\int \frac{1}{\sin x \cdot \cos x} dx$

**3.1.5.** a)  $\int \frac{3}{2 - 5x} dx$

b)  $\int \frac{x}{9 + 4x^2} dx$

**V úlohách 3.2.1. až 3.2.3. vypočítajte neurčité integrály:**

**3.2.1.** a)  $\int \frac{\ln^4 x}{x} dx$

c)  $\int x^2 \cdot e^{x^3} dx$

e)  $\int \frac{x}{(x^2 - 4)^3} dx$

g)  $\int \frac{1}{x^3} \cdot \sin \frac{1}{x^2} dx$

b)  $\int \frac{1}{x^2} \cdot \cos \frac{1}{x} dx$

d)  $\int x \cdot (3x^2 - 4)^5 dx$

f)  $\int x^2 \cdot \sqrt[3]{6 - x^3} dx$

h)  $\int \frac{3x^3}{\sqrt[3]{x^4 + 4}} dx$

**3.2.2.** a)  $\int \sqrt{5 + 2x} dx$

c)  $\int \sin \left( \frac{3x - 5}{2} \right) dx$

b)  $\int \frac{5}{\sqrt[3]{1 - 6x}} dx$

d)  $\int \frac{1}{\sin^2 \left( \frac{x-2}{3} \right)} dx$

e)  $\int \cotg(5x+9) dx$       f)  $\int \frac{3}{\sqrt{(5-2x)^3}} dx$   
 g)  $\int (3-2x)^3 dx$       h)  $\int \frac{1}{(5+3x)^3} dx$

3.2.3. a)  $\int \frac{2^x}{1+4^x} dx$       b)  $\int \frac{x^2}{\sqrt{1-x^6}} dx$   
 c)  $\int \frac{e^x}{x^2} dx$       d)  $\int e^{\cos^2 x} \cdot \sin 2x dx$   
 e)  $\int \frac{\sqrt{1+\ln x}}{x} dx$       f)  $\int \frac{1}{x \cdot \sqrt[3]{\ln 3x}} dx$   
 g)  $\int \frac{\operatorname{tg} \sqrt{x}}{\sqrt{x}} dx$       h)  $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}} dx$

V úlohách 3.3.1. až 3.3.2. vypočítajte neurčité integrály.

3.3.1. a)  $\int x \cdot \sin x dx$       b)  $\int x \cdot e^{2x} dx$   
 c)  $\int (2x-5) \cdot \sin 3x dx$       d)  $\int (5x+2) \cdot 2^x dx$   
 e)  $\int \frac{x}{\sin^2 x} dx$       f)  $\int (x^2 + 6x - 7) \cdot \cos x dx$   
 g)  $\int (4x-x^2) \cdot 5^x dx$       h)  $\int (x^2 + 2x - 3) \cdot e^{-x} dx$   
 i)  $\int (3x+5) \cdot \cos \frac{x}{3} dx$       j)  $\int (2x-7) \cdot \operatorname{tg}^2 x dx$

3.3.2. a)  $\int x^3 \cdot \ln x dx$       b)  $\int \operatorname{arccotg} x dx$   
 c)  $\int \arccos x dx$       d)  $\int \operatorname{arccotg} 2x dx$   
 e)  $\int x \cdot \ln^2 x dx$       f)  $\int \frac{\ln x}{x^3} dx$   
 g)  $\int \arcsin(2x+1) dx$       h)  $\int x^2 \cdot \operatorname{arctg} x dx$

3.3.3. Určte funkciu celkových nákladov  $C(x)$  splňajúcu uvedenú podmienku, ak je daná funkcia marginálnych nákladov  $MC(x)$ .

- |                                       |                     |
|---------------------------------------|---------------------|
| a) $MC(x) = 100 - 2x$                 | $FC = 1\,000$ p. j. |
| b) $MC(x) = 5 + 2x - 0,6x^2$          | $C(5) = 35$ p. j.   |
| c) $MC(x) = \frac{1}{\sqrt{4x+256}}$  | $C(36) = 6$ p. j.   |
| d) $MC(x) = 20 \cdot e^{\frac{x}{2}}$ | $FC = 120$ p. j.    |
| e) $MC(x) = (2x+3) \cdot e^{2x}$      | $FC = 5$ p. j.      |
| f) $MC(x) = 12 + \frac{300}{(x+1)^2}$ | $C(11) = 507$ p. j. |

**3.4.1.** Určte funkciu celkových príjmov  $R(x)$ , ak je daná funkcia marginálnych príjmov  $MR(x)$ .

- |   |  |
|---|--|
| <b>a)</b> $MR(x) = 60 - 2x - 2x^2$<br><b>c)</b> $MR(x) = \frac{2x^2 + 4x + 5}{(x+1)^2}$ | <b>b)</b> $MR(x) = x \cdot e^{\frac{x^2}{2}}$<br><b>d)</b> $MR(x) = \frac{480}{(x+4)^2}$ |
|---|--|

**3.4.2.** Určte funkciu celkových nákladov  $C(x)$  splňajúcu uvedenú podmienku, ak je daná funkcia marginálnych priemerných nákladov  $\tilde{MC}(x)$ .

- |  |   |
|--|---|
| <b>a)</b> $\tilde{MC}(x) = 2 - \frac{1}{x+1}$<br><b>b)</b> $\tilde{MC}(x) = -0,02 - \frac{300}{x^2}$ | $C(1) = 100$ p. j.<br>$C(50) = 450$ p. j. |
|--|---|

**3.4.3.** Určte pri ktoréj produkcii bude výroba rentabilná a pri ktoréj produkcii firma dosiahne maximálny zisk, ak  $MC(x) = 4 + 0,02x$ , fixné náklady sú 50 p. j. a  $MR(x) = 6 - 0,01x$ ,  $x \in \langle 0, 300 \rangle$ .

**3.4.4.** Je daná funkcia marginálnych príjmov  $MR(x)$  a marginálnych nákladov  $MC(x)$ . Určte funkciu celkového zisku, ak

- a)**  $MC(x) = 100 - 0,2x$ ,  $MR(x) = 600 - 0,4x$  a platí, že  $P(100) = 48\ 900$  p. j.
- b)**  $MC(x) = 300 + x$ ,  $MR(x) = 500 - 0,8x$  a platí, že  $C(10) = 3\ 200$  p. j.
- c)**  $MC(x) = 5 - 10x$ ,  $MR(x) = 35 - 4x - 1,5x^2$  a platí, že  $P(4) = 36$  p. j.
- d)**  $MC(x) = 20 + x$ ,  $MR(x) = 40 - 3x$  a platí, že  $FC = 48$  p. j.

**3.4.5.** Vypočítajte obsah rovinného útvaru ohraničeného grafmi funkcií

- a)**  $f_1: y = -x^2 + 2x + 8$ ,  $f_2: y = 0$
- b)**  $f_1: y = 16 - x^2$ ,  $f_2: y = x^2 - 16$
- c)**  $f_1: y = x^2 - 4x + 6$ ,  $f_2: y = -2x^2 + 8x - 3$
- d)**  $f_1: y = x^2 + 6x + 8$ ,  $f_2: y = -x^2 - 10x - 16$
- e)**  $f_1: y = -9 - x^2$  a priamkou  $5x + y + 9 = 0$
- f)**  $f_1: y = x^2$ ,  $f_2: y = 2x^2$ ,  $f_3: y = 1$
- g)**  $f_1: y = 2x^3$ ,  $f_2: y = \frac{x}{2}$
- h)**  $f_1: y = x$ ,  $f_2: y = 3 \cdot \sqrt{x}$

1.3.5) a)  $f_1: y = \frac{\sqrt{x+1}}{x-4}$ ;  $D(f_1): x+1 \geq 0 \wedge x-4 \neq 0$   
 $x \geq -1 \wedge x \neq 4 \Rightarrow D(f_1) = (-1; 4) \cup (4; \infty)$

b)  $f_2: y = \sqrt{\frac{-3}{x^2-5x+4}}$ ;  $D(f) = \frac{-3}{x^2-5x+4} \geq 0$   
 $-3 < 0$  vtedy  
 teda musí platit:  
 $x^2-5x+4 < 0$   
 $(x-4)(x-1) < 0$

$$x^2-5x+4 \neq 0 \quad | \quad D = 25-4 \cdot 4 = 9 \\ x_{1,2} = \frac{5 \pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases} \\ (x-4)(x-1) \neq 0 \quad | \quad x \neq 4 \wedge x \neq 1$$

$ax^2+bx+c = a(x-x_1)(x-x_2)$   
 $a, b, c \in \mathbb{R} \rightarrow$  číslo  
 $x_1, x_2$  - kořeny rovnice

$$(x-4 < 0 \wedge x-1 > 0) \vee (x-4 > 0 \wedge x-1 < 0) \\ (x < 4 \wedge x > 1) \vee (x > 4 \wedge x < 1) \Rightarrow D(f) = (1; 4)$$

c)  $f_3: y = \frac{-3}{\sqrt{x^2-3x}}$ ;  $D(f): x^2-3x \geq 0 \wedge \sqrt{x^2-3x} \neq 0 /^2$   
 $x \cdot (x-3) \geq 0 \wedge x^2-3x \neq 0 \Rightarrow x \cdot (x-3) > 0$

$$(x > 0 \wedge x-3 > 0) \vee (x < 0 \wedge x-3 < 0) \\ (x > 0 \wedge x > 3) \vee (x < 0 \wedge x < 3)$$



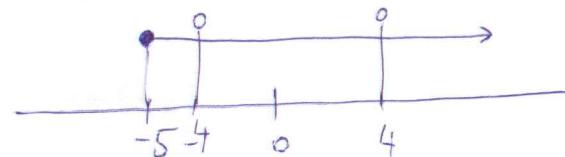
$D(f) = (3; \infty) \cup (-\infty; 0)$

①b

$$\textcircled{1.3.5} \quad \text{d)} \quad f_4: y = \frac{\sqrt{2x+10}}{16-x^2}$$

$$D(f): \begin{aligned} 2x+10 &\geq 0 \quad \wedge \quad 16-x^2 \neq 0 \\ 2x &\geq -10 \quad \wedge \quad (4+x)(4-x) \neq 0 \end{aligned}$$

$$\boxed{x \geq -5} \quad \wedge \quad \boxed{x \neq \pm 4}$$



$$\Rightarrow D(f) = \underline{(-5; -4) \cup (-4; 4) \cup (4; \infty)}$$

$$\textcircled{1.3.6} \quad \text{a)} \quad f_1: y = 4^{\log(2x^2-5x-3)}$$

$$D(f): 2x^2-5x-3 > 0$$

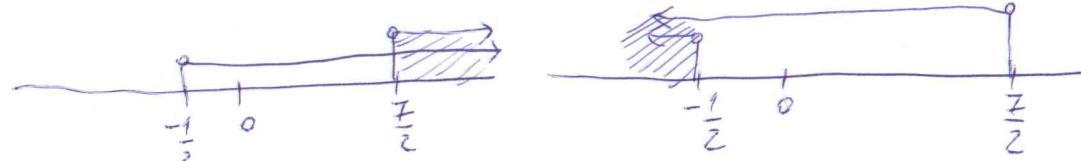
$$D = 25 + 4 \cdot 2 \cdot 3 = 49$$

$$x_{1,2} = \frac{5 \pm \sqrt{49}}{4} = \begin{cases} \frac{14}{4} = \frac{7}{2} \\ -\frac{8}{4} = -\frac{1}{2} \end{cases}$$

$$2 \cdot \left(x - \frac{7}{2}\right) \cdot \left(x + \frac{1}{2}\right) > 0$$

$$\left(x - \frac{7}{2} > 0 \wedge x + \frac{1}{2} > 0\right) \vee \left(x - \frac{7}{2} < 0 \wedge x + \frac{1}{2} < 0\right)$$

$$\left(x > \frac{7}{2} \wedge x > -\frac{1}{2}\right) \vee \left(x < \frac{7}{2} \wedge x < -\frac{1}{2}\right)$$



$$\Rightarrow D(f) = \underline{\left(\frac{7}{2}; \infty\right) \cup \left(-\infty; -\frac{1}{2}\right)}$$

13.5 e)  $f_5: y = \frac{1}{\sqrt{x}} + \sqrt{x^2 - 5} \rightarrow x^2 - 5 \geq 0$  podľa výrovnácia  $(a+b)(a-b) = a^2 - b^2$  možeme napišať:

$$\downarrow \\ x \geq 0 \wedge \sqrt{x} \neq 0$$

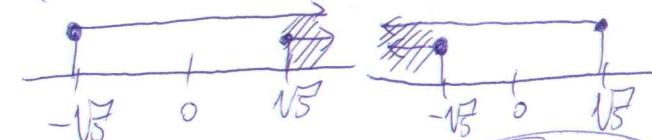
$$\boxed{x > 0}$$

$$(x+\sqrt{5})(x-\sqrt{5}) = x^2 - 5$$

$$\Rightarrow x^2 - 5 \geq 0 \Leftrightarrow_{\text{práve vtedy}} (x+\sqrt{5})(x-\sqrt{5}) \geq 0$$

$$(x+\sqrt{5} \geq 0 \wedge x-\sqrt{5} \geq 0) \vee (x+\sqrt{5} \leq 0 \wedge x-\sqrt{5} \leq 0)$$

$$(x \geq -\sqrt{5} \wedge x \geq \sqrt{5}) \vee (x \leq -\sqrt{5} \wedge x \leq \sqrt{5})$$



$$\boxed{(-\infty; -\sqrt{5})} \quad \boxed{(\sqrt{5}; \infty)}$$

ale zároveň  $\boxed{x > 0}$  a teda  $\underline{\underline{D(f)}} = (\sqrt{5}; \infty)$

f)  $f_6: y = \frac{\sqrt{15+2x-x^2}}{8-2x} \rightsquigarrow 15+2x-x^2 \geq 0 \wedge 8-2x \neq 0$

$$-x^2+2x+15 \geq 0$$

$$D = b^2 - 4ac$$

$$D = 4 - 4 \cdot 15 \cdot (-1)$$

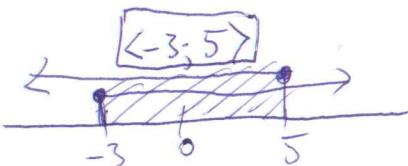
$$D = 64$$

$$x_{1,2} = \frac{-2 \pm \sqrt{64}}{2 \cdot (-1)}$$

$$8 \neq 2x$$

$$2x \neq 8 \quad | :2$$

$$\boxed{x \neq 4}$$



a teda  $\underline{\underline{D(f)}} = (-\infty, -3) \cup (4, 5)$

$$x_{1,2} = \frac{-2 \pm 8}{-2} = \begin{cases} 5 \\ -3 \end{cases}$$

$$\begin{aligned} ax^2 + bx + c &= a(x-x_1)(x-x_2) \\ -x^2 + 2x + 15 &= -(x-5)(x+3) \geq 0 \quad | :(-1) \Rightarrow \text{záporným číslom sa} \\ (x-5)(x+3) &\leq 0 \quad \Rightarrow \text{otáča nerovnosť} \\ (x-5 \leq 0 \wedge x+3 \geq 0) &\vee (x-5 \geq 0 \wedge x+3 \leq 0) \\ (x \leq 5 \wedge x \geq -3) &\vee (x \geq 5 \wedge x \leq -3) \end{aligned}$$

pri násobení a delení

záporným číslom sa otáča nerovnosť

1.3.6

$$b) f_2: y = \ln \sqrt{\frac{3x-1}{x+4}}$$

$$\ln x = \log_e x$$

Plati:

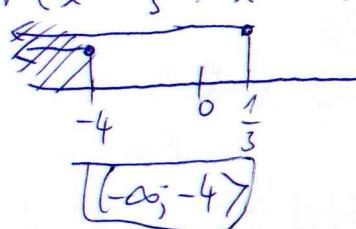
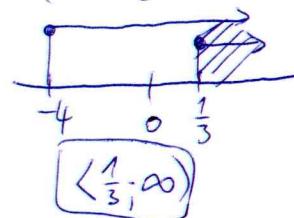
1.  $\sqrt{f(x)} \Rightarrow f(x) \geq 0$
2.  $\frac{1}{f(x)} \Rightarrow f(x) \neq 0$
3.  $\log_a f(x) \Rightarrow f(x) > 0$

(2)

1.  $\sqrt{\frac{3x-1}{x+4}} > 0 \rightsquigarrow$  pre každú hodnotu platí, že je  $\geq 0$   
 Musíme teda vylúčiť iba prípad  $\sqrt{\frac{3x-1}{x+4}} = 0 \Leftrightarrow \frac{3x-1}{x+4} = 0 \Leftrightarrow 3x-1=0 \Leftrightarrow 3x=1 \Leftrightarrow x=\frac{1}{3}$

2.  $x+4 \neq 0$   
 $x \neq -4$

3.  $\frac{3x-1}{x+4} \geq 0 \Leftrightarrow (3x-1 \geq 0 \wedge x+4 \geq 0) \vee (3x-1 \leq 0 \wedge x+4 \leq 0)$   
 $(x \geq \frac{1}{3} \wedge x \geq -4) \vee (x \leq \frac{1}{3} \wedge x \leq -4)$

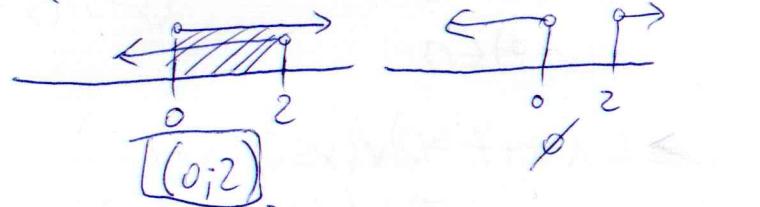
všetko dokopy:  $x \neq \frac{1}{3}$  $x \neq -4$  $x \in (-\infty; -4) \cup \left(\frac{1}{3}; \infty\right)$ 

$$D(f) = (-\infty; -4) \cup \left(\frac{1}{3}; \infty\right)$$

103.6 c)  $f_3: y = \frac{-1}{\ln(2x-x^2)}$   $\Rightarrow 2x-x^2 > 0 \wedge \ln(2x-x^2) \neq 0 \wedge x(2-x) > 0 \wedge 2x-x^2 \neq 1$   $\Rightarrow \boxed{\ln X = 0 \Leftrightarrow X = 1}$ !

$$(x > 0 \wedge 2-x > 0) \vee (x < 0 \wedge 2-x < 0) \wedge (-x^2+2x-1 \neq 0) / \cdot (-1)$$

$$(x > 0 \wedge x < 2) \vee (x < 0 \wedge x > 2) \wedge (x^2-2x+1 \neq 0)$$



$$D = 4 - 4 = 0$$

$$x_{1,2} = \frac{2 \pm 0}{2} = 1$$

$$x^2 - 2x + 1 = (x-1)(x-1) = (x-1)^2 \neq 0$$

$\boxed{x \neq 1}$

$$\underline{D(f) = (0; 1) \cup (1; 2)}$$

d)  $f_4: y = \log x^2 + \log(4-x^2)$

$$x^2 > 0 \wedge 4-x^2 > 0$$

↓

platí vždy okrem

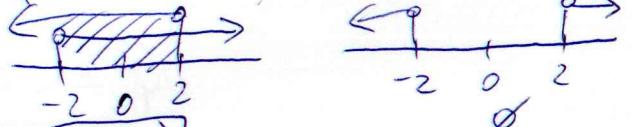
pripadu  $\boxed{x \neq 0}$

(kdečé číslo umocnené  
na 2. je kladné alebo 0)

$$(2-x)(2+x) > 0$$

$$(2-x > 0 \wedge 2+x > 0) \vee (2-x < 0 \wedge 2+x < 0)$$

$$(x < 2 \wedge x > -2) \vee (x > 2 \wedge x < -2)$$



$$\underline{D(f) = (-2; 0) \cup (0; 2)}$$

(1.3.6.) e)  $f_5: y = \sqrt{1 - \log(x^2 + 7x + 10)}$

$$\Rightarrow 1 - \log(x^2 + 7x + 10) \geq 0 \quad \wedge \quad x^2 + 7x + 10 > 0$$

$$\log(x^2 + 7x + 10) \leq 1 \quad \wedge \quad D = 49 - 4 \cdot 10 = 9$$

$$x^2 + 7x + 10 \leq 10^1 / 10$$

$$x_{1,2} = \frac{-7 \pm 3}{2} = \begin{cases} -5 \\ -2 \end{cases}$$

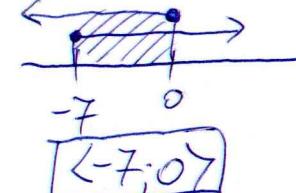
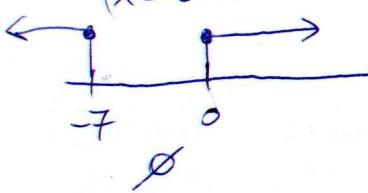
$$x^2 + 7x \leq 0$$

$$x(x+7) \leq 0$$

$$(x+5)(x+2) > 0$$

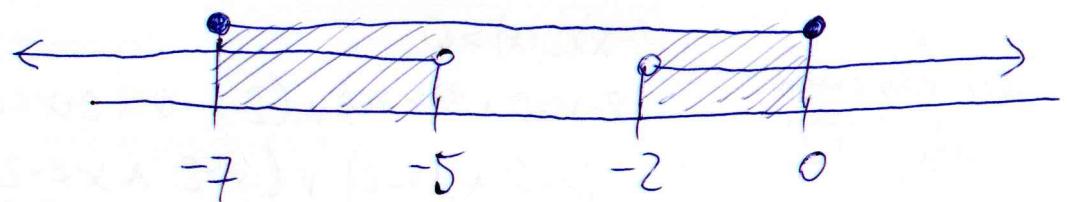
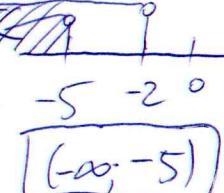
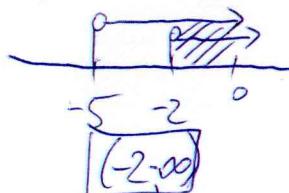
$$(x \geq 0 \wedge x+7 \leq 0) \vee (x \leq 0 \wedge x+7 \geq 0)$$

$$(x \geq 0 \wedge x \leq -7) \vee (x \leq 0 \wedge x \geq -7)$$



$$(x+5 > 0 \wedge x+2 > 0) \vee (x+5 \leq 0 \wedge x+2 \leq 0)$$

$$(x > -5 \wedge x > -2) \vee (x < -5 \wedge x < -2)$$



$$\underline{\underline{D(f) = (-7, -5) \cup (-2, 0)}}$$

(4)

(5)

1.3.6 f)

$$f_6: y = \frac{\sqrt{\ln(x-1)}}{x-2}$$

$$\ln(x-1) \geq 0 \wedge x-1 > 0 \wedge x-2 \neq 0$$

$$\log_e(x-1) \geq 0 \wedge x > 1 \wedge x \neq 2$$

$$x-1 \geq e^0$$

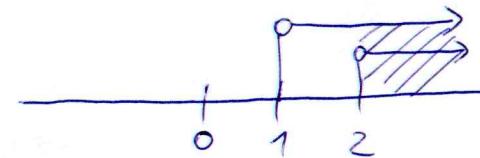
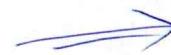
$$(1; \infty)$$

$$x-1 \geq 1$$

$$x \geq 2$$

$$(2; \infty)$$

!  $x=1$   
pre všetky  $x$  !



$$\underline{\underline{D(f) = (2; \infty)}}$$

1.3.7 a)

$$f_7: y = \arcsin \frac{2x+4}{x}$$

$$\boxed{x \neq 0}$$

~~$$\begin{aligned} -1 &\leq \frac{2x+4}{x} \leq 1 \\ x &\leq 2x+4 \leq x \end{aligned}$$~~

Dalsie podmienky pre Definičný obor (arcusy):

$$4. \arcsin X \Rightarrow X \in [-1; 1]$$

$$5. \arccos X \Rightarrow X \in [-1; 1]$$

$$\frac{2x+4}{x} \leq 1 / -1 \quad \wedge \quad \frac{2x+4}{x} \geq -1 / +1$$

$$\frac{2x+4}{x} - 1 \leq 0 \quad \wedge \quad \frac{2x+4}{x} + 1 \geq 0$$

$$\frac{2x+4}{x} - \frac{x}{x} \leq 0 \quad \wedge \quad \frac{2x+4}{x} + \frac{x}{x} \geq 0$$

$$\frac{2x+4-x}{x} \leq 0 \quad \wedge \quad \frac{2x+4+x}{x} \geq 0$$

$$\frac{x+4}{x} \leq 0 \quad \wedge \quad \frac{3x+4}{x} \geq 0$$

$$(x+4 \leq 0 \wedge x \geq 0) \vee (x+4 \geq 0 \wedge x \leq 0)$$

$$(x \leq -4 \wedge x \geq 0) \vee (x \geq -4 \wedge x \leq 0)$$

$$\begin{array}{c} \bullet \rightarrow \\ -4 \\ \emptyset \\ 0 \end{array} \quad \begin{array}{c} \bullet \rightarrow \\ -4 \\ \emptyset \\ 0 \end{array} \quad \begin{array}{c} \bullet \leftarrow \\ -4 \\ \emptyset \\ 0 \end{array}$$

$$(3x+4 \geq 0 \wedge x \geq 0) \vee (3x+4 \leq 0 \wedge x \leq 0)$$

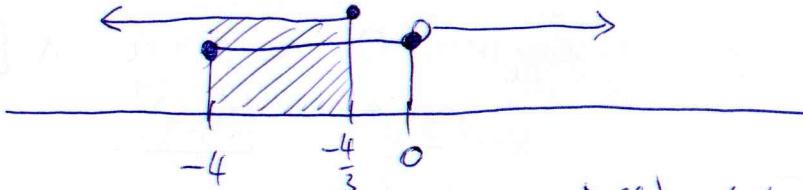
$$(x \geq -\frac{4}{3} \wedge x \geq 0) \vee (x \leq -\frac{4}{3} \wedge x \leq 0)$$

$$\begin{array}{c} \bullet \rightarrow \\ -\frac{4}{3} \\ \emptyset \\ 0 \end{array} \quad \begin{array}{c} \bullet \leftarrow \\ -\frac{4}{3} \\ \emptyset \\ 0 \end{array} \quad \begin{array}{c} \bullet \leftarrow \\ -\frac{4}{3} \\ \emptyset \\ 0 \end{array}$$

dohromady:  $x \neq 0$

$$\langle -4; 0 \rangle$$

$$\langle 0; \infty \rangle \cup \left(-\infty; -\frac{4}{3}\right)$$



$$\underline{\underline{D(f) = \langle -4; -\frac{4}{3} \rangle}}$$

(13.7) b)  $f_2: y = \operatorname{arccos} \frac{x^2}{x^2 - 2}$

chtátk! pre arctg a arccos neplatia  
žiadne obmedzenia pre def. dom!

$$x^2 - 2 \neq 0$$

$$x^2 \neq 2 \Rightarrow x \neq \pm \sqrt{2} \Rightarrow \underline{\underline{D(f) = (-\infty; -\sqrt{2}) \cup (-\sqrt{2}; \sqrt{2}) \cup (\sqrt{2}, \infty)}}$$

c)  $f_3: y = \arccos\left(\frac{1}{x^2}\right)$

$$x^2 \neq 0 \Leftrightarrow \boxed{x \neq 0}$$

$$\frac{1}{x^2} \leq 1 \quad \wedge \quad \frac{1}{x^2} \geq -1$$

$$\frac{1}{x^2} - 1 \leq 0 \quad \wedge \quad \frac{1}{x^2} + 1 \geq 0$$

$$\frac{1}{x^2} - \frac{x^2}{x^2} \leq 0 \quad \wedge \quad \frac{1}{x^2} + \frac{x^2}{x^2} \geq 0$$

$$\frac{1-x^2}{x^2} \leq 0 \quad \wedge \quad \frac{1+x^2}{x^2} \geq 0$$

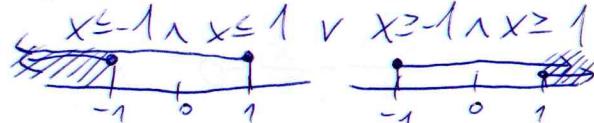
z prvej podmienky vieme, že  $x \neq 0$   
a tiež platí  $x^2 \geq 0$  všetky (druhá mocnina nikdy nie je záporná)

$\frac{1+x^2}{x^2} \geq 0$  teda platí všetky, lebo  $\frac{1+ \text{kladné č.}}{\text{kladné č.}}$  je všetky kladné

Dalej,  $\frac{1-x^2}{x^2} \leq 0 \Leftrightarrow 1-x^2 \leq 0$  (lebo v menovateli je  $x^2 \geq 0$ )

podľa vzorca  $a^2 - b^2 = (a+b)(a-b)$  platí:  $1-x^2 \leq 0 \Leftrightarrow (1+x)(1-x) \leq 0$   
 $(1+x \leq 0 \wedge 1-x \geq 0) \vee (1+x \geq 0 \wedge 1-x \leq 0)$

$$\underline{\underline{D(f) = (-\infty; -1) \cup (1; \infty)}}$$



163.7 d)

$$f_4: y = \frac{1}{x} + \arccos(x^2 - 1)$$

$$\boxed{x \neq 0}$$

$$x^2 - 1 \leq 1$$

$$x^2 \leq 2$$

$$x^2 \geq 0$$

$$(x+1)(x-1) \leq 0$$

$$(x+\sqrt{2} \leq 0 \wedge x-\sqrt{2} \geq 0) \vee (x+\sqrt{2} \geq 0 \wedge x-\sqrt{2} \leq 0)$$

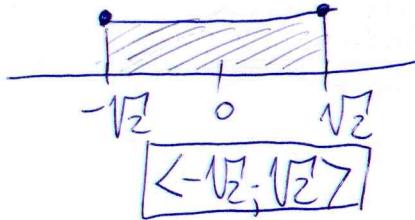
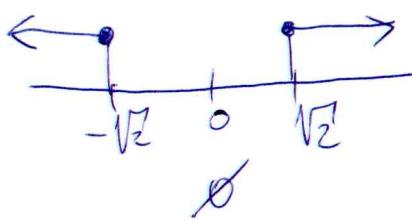
$$(x \leq -\sqrt{2} \wedge x \geq \sqrt{2}) \vee (x \geq -\sqrt{2} \wedge x \leq \sqrt{2})$$

$\wedge$

$$x^2 - 1 \geq -1 \Leftrightarrow$$

$$x^2 \geq 0 \rightarrow \text{platí vždy}$$

$$\Rightarrow \underline{D(f) = (-\sqrt{2}; 0) \cup (0; \sqrt{2})}$$



$$e) f_5: y = \frac{x}{\arctg(12-4x)}$$

$$\arctg(12-4x) \neq 0$$

$$12-4x \neq 0$$

$$\boxed{x \neq 3}$$

platí:  $\arctg X = 0 \Leftrightarrow X = 0$

(dá sa najst v tabuľkach)

$$\underline{D(f) = (-\infty; 3) \cup (3; \infty)}$$

1.3.7

$$f_6: y = \sqrt{\arcsin(x-4)}$$

$$-1 \leq x-4 \leq 1 \quad |+4 \quad \wedge \quad \arcsin(x-4) \geq 0$$

$$3 \leq x \leq 5$$

$$\boxed{[3;5]}$$

môžeme pôčítať  
takto náraz, ak tam

môžeme iba x a nie  $x^2$   
a ak x nie je v menovateli

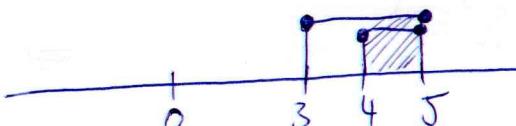
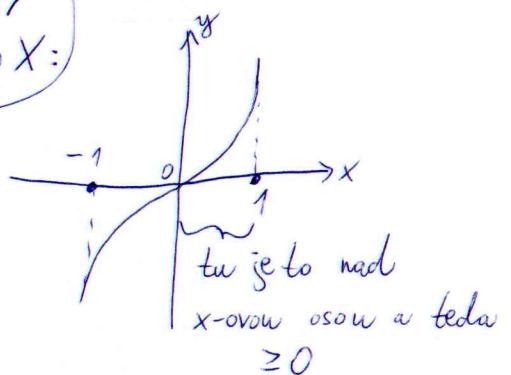
$$0 \leq x-4 \leq 1 \quad |+4$$

$$4 \leq x \leq 5$$

$$\boxed{[4;5]}$$

$$\arcsin X \geq 0 \Leftrightarrow X \in [0; 1]$$

toto vidieť = grafu  $\arcsin X$ :



$$\underline{\underline{D(f) = [4;5]}}$$

1.3.8

$$a) f_7: y = \log(1-2x) - 3\arcsin \frac{3x-1}{2}$$

$$1-2x > 0 \quad \wedge \quad -1 \leq \frac{3x-1}{2} \leq 1 \quad | \cdot 2$$

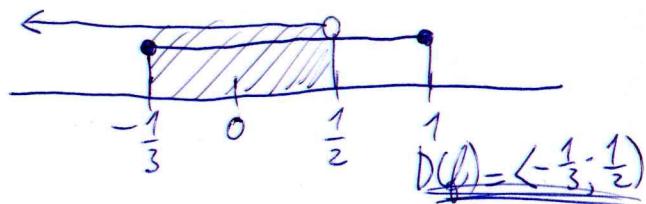
$$\boxed{x < \frac{1}{2}}$$

$$-2 \leq 3x-1 \leq 2 \quad |+1$$

$$-1 \leq 3x \leq 3 \quad |:3$$

$$-\frac{1}{3} \leq x \leq 1$$

$$\boxed{[-\frac{1}{3}; 1]}$$



$$\underline{\underline{D(f) = [-\frac{1}{3}; \frac{1}{2}]}}$$

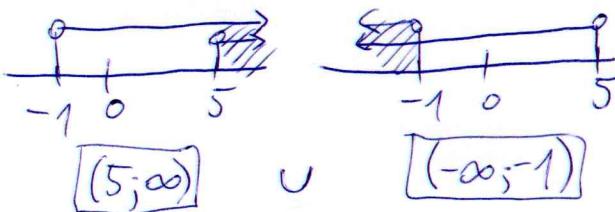
(103.8) b)

$$f_2: y = 5 \cdot \log\left(\frac{x+1}{x-5}\right) - \frac{\sqrt{5x-10}}{x^2-36}$$

$$\frac{x+1}{x-5} > 0 \quad \wedge \quad x-5 \neq 0 \quad \wedge \quad 5x-10 \geq 0 \quad \wedge \quad x^2-36 \neq 0$$

$$(x+1>0 \wedge x-5>0) \vee (x+1<0 \wedge x-5<0) \quad \boxed{x \neq 5}$$

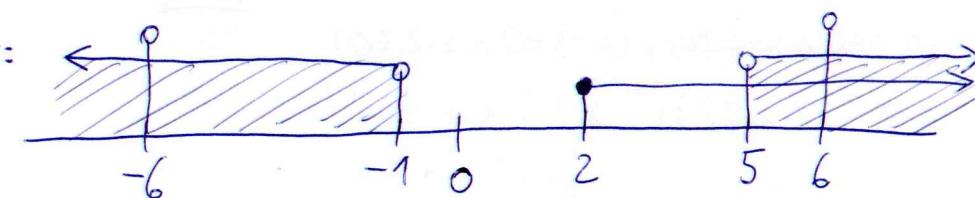
$$(x>-1 \wedge x>5) \vee (x<-1 \wedge x<5)$$



$$5x \geq 10 \quad \wedge \quad (x-6)(x+6) \neq 0$$

$\boxed{x \geq 2}$   $\quad \quad \quad \boxed{x \neq \pm 6}$

všechno dokopy:



$$D(f) = (-\infty; -6) \cup (-6; -1) \cup (5; 6) \cup (6; \infty)$$

$$0) f_3: y = \arccos(3+2x) + \sqrt{\frac{x-2}{x+3}}$$

$$-1 \leq 3+2x \leq 1 \quad | -3 \quad \wedge \quad x+3 \neq 0$$

$$-4 \leq 2x \leq -2 \quad | :2 \quad \boxed{x \neq -3}$$

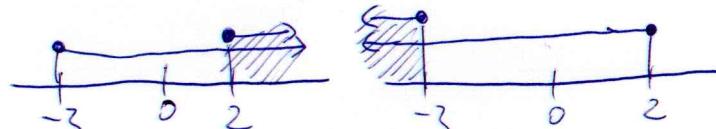
$$-2 \leq x \leq -1$$

$$\boxed{[-2; -1]}$$

$$\wedge \quad \frac{x-2}{x+3} \geq 0$$

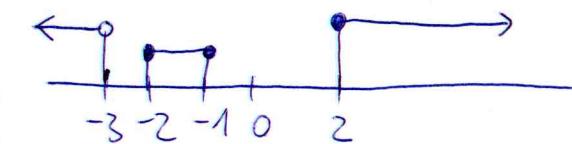
$$(x-2 \geq 0 \wedge x+3 \geq 0) \vee (x-2 \leq 0 \wedge x+3 \leq 0)$$

$$(x \geq 2 \wedge x \geq -3) \vee (x \leq 2 \wedge x \leq -3)$$



$$\boxed{(-\infty; -3] \cup [2; \infty)}$$

Doklomady:



$$\boxed{D(f) = \emptyset}$$

(9)

10

103.8 d)  $f_4: y = \frac{\sqrt{x^2 - 5x + 6}}{\ln(2x-5)} - \sqrt{5-x}$

$$x^2 - 5x + 6 \geq 0 \quad \wedge \quad \ln(2x-5) \neq 0 \quad \wedge \quad 2x-5 > 0 \quad \wedge \quad 5-x \geq 0$$

$$D = 25 - 24 = 1$$

$$x_{1,2} = \frac{5 \pm 1}{2} = \begin{cases} 3 \\ 2 \end{cases}$$

$$(x-3)(x-2) \geq 0$$

$$(x-3 \geq 0 \wedge x-2 \geq 0) \vee (x-3 \leq 0 \wedge x-2 \leq 0)$$

$$(x \geq 3 \wedge x \geq 2) \vee (x \leq 3 \wedge x \leq 2)$$



$$\boxed{[3; \infty) \cup (-\infty; 2]}$$

$$\log_e(2x-5) \neq 0$$

$$2x-5 \neq e^0$$

$$2x-5 \neq 1$$

$$\begin{array}{c} 2x \neq 6 \\ \boxed{x \neq 3} \end{array}$$

$$2x > 5$$

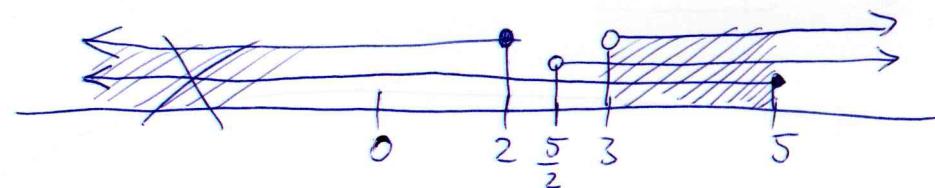
$$x > \frac{5}{2}$$

$$\boxed{(\frac{5}{2}; \infty)}$$

$$\nexists x \leq 5$$

$$\boxed{(-\infty; 5)}$$

Dohromady:



$$\underline{D(f) = (3; 5)}$$

(13.8)

$$e) f_5: y = \frac{\sqrt{x^2 - x - 2}}{\ln x} - 4 \cdot \arcsin \frac{1-2x}{4}$$

$$x^2 - x - 2 \geq 0 \wedge \ln x \neq 0$$

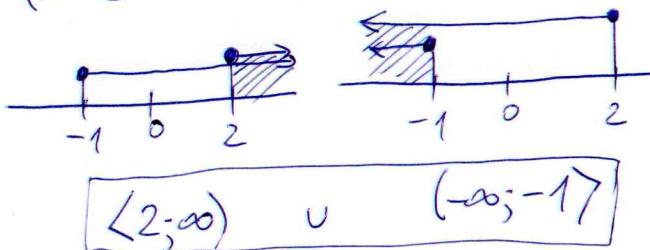
$$D = 1+4 \cdot 2 = 9$$

$$x_{1,2} = \frac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$$

$$(x-2)(x+1) \geq 0$$

$$(x-2 \geq 0 \wedge x+1 \geq 0) \vee (x-2 \leq 0 \wedge x+1 \leq 0)$$

$$(x \geq 2 \wedge x \geq -1) \vee (x \leq 2 \wedge x \leq -1)$$



$$x > 0 \wedge -1 \leq \frac{1-2x}{4} \leq 1 \quad | \cdot 4$$

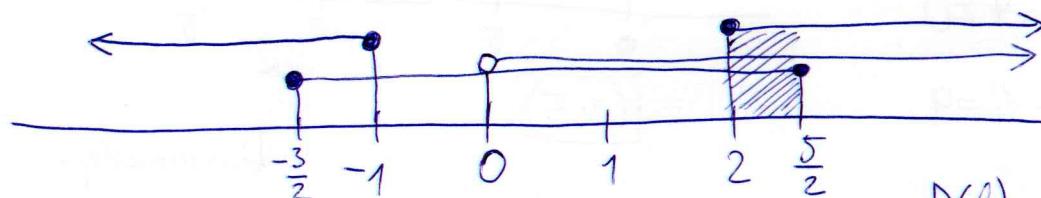
$$-4 \leq 1-2x \leq 4 \quad | -1$$

$$-5 \leq -2x \leq 3 \quad | :(-2)$$

$$\frac{5}{2} \geq x \geq -\frac{3}{2}$$

$$\left\langle -\frac{3}{2}; \frac{5}{2} \right\rangle$$

Dohromady:



$$\underline{D(f) = \left\langle 2; \frac{5}{2} \right\rangle}$$

(11)

1.3.8

$$f_6: y = \sqrt{\log \frac{5x-x^2}{4}} + \frac{1}{\log_2 x - 2}$$

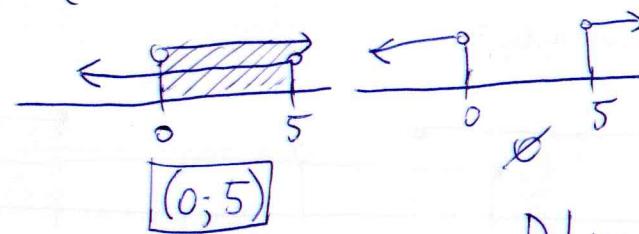
$$\log \frac{5x-x^2}{4} \geq 0 \quad \wedge \quad \frac{5x-x^2}{4} > 0 \quad \wedge \quad \log_2 x - 2 \neq 0 \quad \wedge \quad x > 0$$

$$\frac{5x-x^2}{4} \geq 10^0 \quad \uparrow \quad \log_2 x \neq 2 \quad (0; \infty)$$

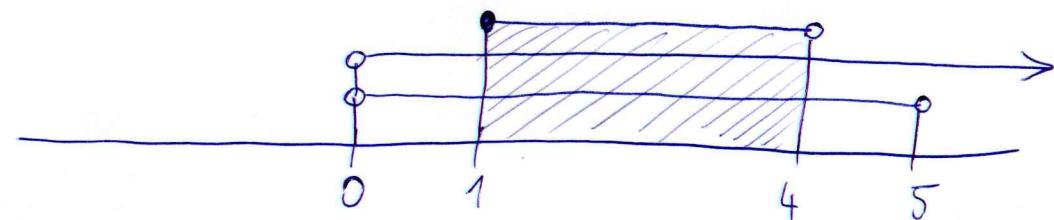
$$\frac{5x-x^2}{4} \geq 1/4 \quad x(5-x) > 0 \quad x \neq 4$$

$$(x > 0 \wedge 5-x > 0) \vee (x < 0 \wedge 5-x < 0)$$

$$(x > 0 \wedge x < 5) \vee (x < 0 \wedge x > 5)$$

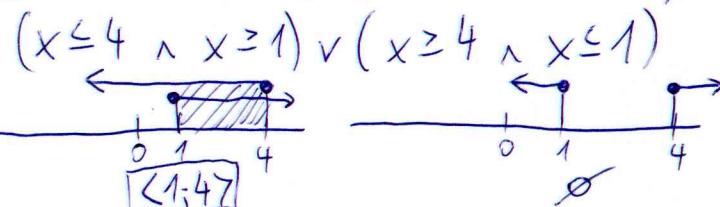


Dohromady:



$$\underline{D(f) = [1; 4]}$$

$$(x-4 \leq 0 \wedge x-1 \geq 0) \vee (x-4 \geq 0 \wedge x-1 \leq 0)$$



(13.8)

$$g) f \neq: y = \arccos \frac{3}{2x-5} + \ln(6+11x-2x^2)$$

$$\frac{3}{2x-5} \geq -1$$

$$\wedge \quad \frac{3}{2x-5} \leq 1$$

$$\wedge \quad -2x^2 + 11x + 6 > 0$$

$$\wedge \quad 2x-5 \neq 0$$

$$\frac{3}{2x-5} + 1 \geq 0$$

$$\frac{3}{2x-5} - 1 \leq 0$$

$$D = 121 + 4 \cdot 2 \cdot 6 = 169$$

$$x_{1,2} = \frac{-11 \pm \sqrt{169}}{-4} = \begin{cases} -\frac{1}{2} \\ 6 \end{cases}$$

$$\frac{3}{2x-5} + \frac{2x-5}{2x-5} \geq 0$$

$$\frac{3}{2x-5} - \frac{2x-5}{2x-5} \leq 0$$

$$-2(x-6)(x+\frac{1}{2}) > 0 \quad /:(-2)$$

$$\frac{3+2x-5}{2x-5} \geq 0$$

$$\frac{3-2x+5}{2x-5} \leq 0$$

$$(x-6)(x+\frac{1}{2}) < 0$$

$$\frac{2x-2}{2x-5} \geq 0 \quad /:2$$

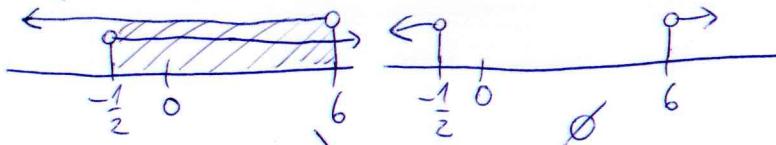
$$\frac{8-2x}{2x-5} \leq 0 \quad /:2$$

$$(x-6 \leq 0 \wedge x+\frac{1}{2} > 0) \vee (x-6 > 0 \wedge x+\frac{1}{2} < 0)$$

$$\frac{x-1}{2x-5} \geq 0$$

$$\frac{4-x}{2x-5} \leq 0$$

$$(x < 6 \wedge x > -\frac{1}{2}) \vee (x > 6 \wedge x < -\frac{1}{2})$$



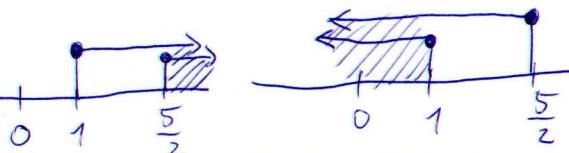
$$(-\frac{1}{2}; 6)$$

$$(x-1 \geq 0 \wedge 2x-5 \geq 0) \vee (x-1 \leq 0 \wedge 2x-5 \leq 0)$$

$$(4-x \leq 0 \wedge 2x-5 \geq 0) \vee (4-x \geq 0 \wedge 2x-5 \leq 0)$$

$$(x \geq 1 \wedge x \geq \frac{5}{2}) \vee (x \leq 1 \wedge x \leq \frac{5}{2})$$

$$(x \geq 4 \wedge x \geq \frac{5}{2}) \vee (x \leq 4 \wedge x \leq \frac{5}{2})$$

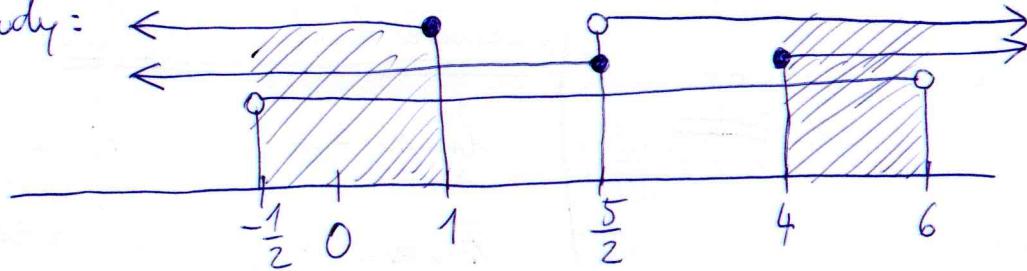


$$\left(\frac{5}{2}; \infty\right) \cup (-\infty; 1]$$

$$\left(4; \infty\right) \cup \left(-\infty; \frac{5}{2}\right)$$

(13)

Dohromady:



$$D(f) = \left(-\frac{1}{2}; 1\right] \cup <4; 6)$$

1.3.8

h)  $f_8: y = \frac{14-x}{\ln(x^2-4)} + \arccos(3x+7)$

$$\ln(x^2-4) \neq 0 \wedge x^2-4 > 0 \wedge -1 \leq 3x+7 \leq 1 \rightsquigarrow -8 \leq 3x \leq -6 \quad | :3$$

$$x^2-4 \neq 1 \quad (x-2)(x+2) > 0$$

$$x^2 \neq 5$$

$$x \neq \pm\sqrt{5}$$

$$(x-2 > 0 \wedge x+2 > 0) \vee (x-2 < 0 \wedge x+2 < 0)$$

$$(x > 2 \wedge x > -2) \vee (x < 2 \wedge x < -2)$$

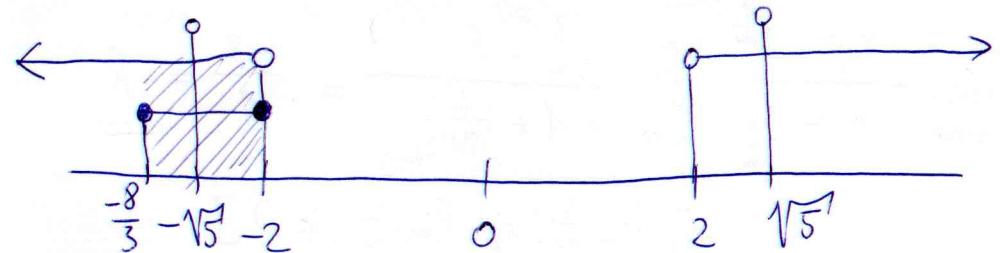


$$(2; \infty) \cup (-\infty; -2)$$

$$-\frac{8}{3} \leq x \leq -2$$

$$\left[-\frac{8}{3}; -2\right]$$

Dohromady:



$$D(f) = \left[-\frac{8}{3}; -15\right] \cup (-15; -2)$$

2.2.1

$$\lim_{n \rightarrow \infty} (-2n^5 + 3n^2 - 10) = \lim_{n \rightarrow \infty} n^5 \cdot \left( -2 + \frac{3}{n^3} - \frac{10}{n^5} \right) = \infty \cdot (-2) = -\infty$$

$\underbrace{\lim_{n \rightarrow \infty} ( )}_{= -2}$

limita závorky je -2, protože  $\lim_{n \rightarrow \infty} \frac{3}{n^3} = 3 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^3} = 3 \cdot 0 = 0$

$$\lim_{n \rightarrow \infty} \frac{10}{n^5} = 10 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^5} = 0$$

[lebo  $\frac{1}{\infty^5} = \frac{1}{\infty} = 0$ ]

$$\begin{aligned} \lim_{n \rightarrow \infty} \left( 3n^4 - \frac{3}{n^3} + 4n \right) &= \lim_{n \rightarrow \infty} 3n^4 - \lim_{n \rightarrow \infty} \frac{3}{n^3} + \lim_{n \rightarrow \infty} 4n = \\ &= 3 \cdot \lim_{n \rightarrow \infty} n^4 - 3 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^3} + 4 \lim_{n \rightarrow \infty} n = 3 \cdot \infty - 3 \cdot 0 + 4 \cdot \infty = \infty \end{aligned}$$

$$\lim_{n \rightarrow \infty} \frac{3-8n}{n+2} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\left(\frac{3}{n}\right) - 8}{1 + \left(\frac{2}{n}\right)} = \frac{-8}{1} = -8$$

[ $\lim_{n \rightarrow \infty} \frac{3}{n}$  aj  $\lim_{n \rightarrow \infty} \frac{2}{n}$  je 0]

$$\lim_{n \rightarrow \infty} \frac{4n^2}{-n+2} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{(4n)^2}{-1 + \left(\frac{2}{n}\right)} = \frac{\infty}{-1} = -\infty$$

[čitatela aj menovatela delíme  
na nejväčšiu možnosť n v menovateli]

### Základné limity a vzťahy:

$$\lim_{n \rightarrow \infty} n = \infty$$

$$\lim_{n \rightarrow \infty} n = c$$

$$\lim_{n \rightarrow \infty} n = -\infty$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$$\lim_{n \rightarrow -\infty} \frac{1}{n} = 0$$

$$\lim_{n \rightarrow 0} \frac{1}{n} \quad ? \quad (\text{neexistuje})$$

$$\infty \cdot \infty = \infty$$

$$\sqrt[n]{\infty} = \infty$$

$$\infty \cdot (-\infty) = -\infty \cdot \infty = -\infty$$

$$c^{-\infty} = \frac{1}{c^\infty} = 0$$

$$-\infty \cdot (-\infty) = \infty$$

$$\forall c > 0$$

$$\infty + \infty = \infty$$

$$-\infty - \infty = -\infty$$

$$c \cdot \infty = \begin{cases} -\infty & \text{ak } c < 0 \\ \infty & \text{ak } c > 0 \\ ? & \text{ak } c = 0 \text{ (treba zistiť)} \end{cases}$$

$$\infty + c = \infty \quad \text{pre všetky čísla } c \\ (\text{t.j. } \forall c \in \mathbb{R})$$

$$\infty^\infty = \infty$$

$$\infty^\circ = ?$$

$$0 \cdot \infty = ?$$

$$\infty - \infty = ?$$

môže byť ľubovoľný

(treba zistiť v konkrétnom prípade)

(2.2.1) e)  $\lim_{n \rightarrow \infty} \frac{7n + 2n^3}{7n^2 - n - 1} \cdot \frac{1}{n^2} = \lim_{n \rightarrow \infty} \frac{\frac{7}{n} + 2n^3}{7 - \frac{1}{n} - \frac{1}{n^2}} = \frac{\infty}{7} = \infty$

f)  $\lim_{n \rightarrow \infty} \frac{-3n^2 + 4n}{4n^3 - n + 2} \cdot \frac{1}{n^3} = \lim_{n \rightarrow \infty} \frac{\frac{-3n^2}{n^3} + \frac{4n}{n^3}}{\frac{4n^3}{n^3} - \frac{n}{n^3} + \frac{2}{n^3}} = \lim_{n \rightarrow \infty} \frac{\frac{-3}{n} + \frac{4}{n^2}}{4 - \frac{1}{n^2} + \frac{2}{n^3}} = \frac{0}{4} = 0$

g)  $\lim_{n \rightarrow \infty} \left( \frac{3n - 5}{4 + 3n} \cdot \frac{1}{n} \right)^5 = \lim_{n \rightarrow \infty} \left( \frac{3 - \frac{5}{n}}{\frac{4}{n} + 3} \right)^5 = \left( \frac{3}{3} \right)^5 = 1^5 = 1$

|                             |
|-----------------------------|
| $(a+b)(a-b) = a^2 - b^2$    |
| $(a+b)^2 = a^2 + 2ab + b^2$ |
| $(a-b)^2 = a^2 - 2ab + b^2$ |

h)  $\lim_{n \rightarrow \infty} \left( \frac{5n - n^5}{2n^5 + n} \cdot \frac{1}{n^5} \right)^4 = \lim_{n \rightarrow \infty} \left( \frac{\frac{5}{n^4} - 1}{2 + \frac{1}{n^4}} \right)^4 = \left( \frac{-1}{2} \right)^4 = \frac{1}{16}$

i)  $\lim_{n \rightarrow \infty} \frac{(n+3)(2n-1)}{(n-3)^2} = \lim_{n \rightarrow \infty} \frac{2n^2 - n + 6n - 3}{n^2 - 6n + 9} = \lim_{n \rightarrow \infty} \frac{2n^2 + 5n - 3}{n^2 - 6n + 9} \cdot \frac{1}{n^2} = \lim_{n \rightarrow \infty} \frac{2 + \frac{5}{n} - \frac{3}{n^2}}{1 - \frac{6}{n} + \frac{9}{n^2}} = \frac{2}{1} = 2$

j)  $\lim_{n \rightarrow \infty} \sqrt{\frac{9+n^2}{4n^2}} = \lim_{n \rightarrow \infty} \sqrt{\frac{\frac{9}{n^2} + \frac{n^2}{4n^2}}{1}} = \lim_{n \rightarrow \infty} \sqrt{\frac{\frac{9}{4n^2} + \frac{1}{4}}{1}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

k)  $\lim_{n \rightarrow \infty} \frac{3n+2}{\sqrt[4]{16n^4 + 6n^3 - 2}} \cdot \frac{1}{n^2} = \lim_{n \rightarrow \infty} \frac{\frac{3}{n} + \frac{2}{n^2}}{\sqrt[4]{16n^4 + 6n^3 - 2}} = \lim_{n \rightarrow \infty} \frac{\frac{3}{n} + \frac{2}{n^2}}{\sqrt[4]{\frac{16n^4}{n^4} + \frac{6n^3}{n^4} - \frac{2}{n^4}}} = \lim_{n \rightarrow \infty} \frac{\frac{3}{n} + \frac{2}{n^2}}{\sqrt[4]{16 + \frac{6}{n} - \frac{2}{n^3}}} = \frac{0}{\sqrt[4]{16}} = \frac{0}{4} = 0$

[nejvýčisťacia mocnina v menovateli je  $n^4$ , ale je  $n^2$  podíl, teda delíme  $\sqrt[4]{n^4} = n$ ]

2.2.4

$$a) \lim_{n \rightarrow \infty} \left(1 + \frac{1}{5n}\right)^{2n+6}$$

→ pre výpočet tohto druhu limit sa využíva vzorec:

$$\boxed{\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e}$$

- namiesto „n“ vo vzoreci môže byť ľakotiek čo obsahuje „n“;  
podmienkou je, aby v menovateli vo mŕtvi v závorku a hore  
v exponente vystupoval ten istý výraz

- našou úlohou potom je upraviť pôvodnú limitu na takúto limitu  
a následne vypočítať limitu exponentu (toto, na čo je umocnené „e“),

pretože platí:

$$\lim_{n \rightarrow \infty} \left[ \left(1 + \frac{1}{n}\right)^n \right]^x = e^{\lim_{n \rightarrow \infty} x}$$

→ ukážme si to na tomto príklade:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{5n}\right)^{2n+6} = \lim_{n \rightarrow \infty} \left[ \left(1 + \frac{1}{5n}\right)^{5n} \right]^{\frac{2n+6}{5n}} = \cancel{e^{\lim_{n \rightarrow \infty} \frac{2n+6}{5n}}} = \underline{\underline{e^{\frac{2}{5}}}}$$

shádzime sa dostať  
kvôd  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{f(n)}\right)^{f(n)}$

↓  
umocňujeme teda závorku na  
výraz, kt. je v menovateli.  
To celé je potom umocnené na  
pôvodnú mocninu (2n+6) ale este  
lomeno „5n“, lebo to sme tam  
umelo „pridalí“

túto limitu  
vypočítame  
zvlášť:

$$\lim_{n \rightarrow \infty} \frac{2n+6}{5n \cdot \frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{2 + \frac{6}{n}}{5} = \underline{\underline{\frac{2}{5}}}$$

2.2.4

$$b) \lim_{n \rightarrow \infty} \left(1 + \frac{7}{3n}\right)^{n-1} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{\frac{3n}{7}}\right)^{\frac{3n}{7}}\right]^{\frac{n-1}{\frac{3n}{7}}} = e^{\lim_{\infty} \frac{n-1}{\frac{3n}{7}}} = e^{\frac{7}{3}}$$

$$\lim_{n \rightarrow \infty} \frac{n-1}{\frac{3n}{7}} = \lim_{\infty} \frac{n-1}{\frac{3n}{7}} = \lim_{\infty} \frac{7 \cdot (n-1)}{3n} = \lim_{n \rightarrow \infty} \frac{7n-7}{3n} \cdot \frac{1}{n} = \lim_{\infty} \frac{7 - \frac{7}{n}}{3} \xrightarrow{0} = \frac{7}{3}$$

Plauti:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}$$

$$c) \lim_{n \rightarrow \infty} \left(\frac{n+5}{n+4}\right)^{2n-1} = \lim_{n \rightarrow \infty} \left(\frac{n+4+1}{n+4}\right)^{2n-1} = \lim_{\infty} \left(\frac{n+4}{n+4} + \frac{1}{n+4}\right)^{2n-1} = \lim_{\infty} \left[\left(1 + \frac{1}{n+4}\right)^{n+4}\right]^{\frac{2n-1}{n+4}} = e^2$$

$$\lim_{n \rightarrow \infty} \frac{\frac{2n-1}{n}}{\frac{n+4}{n} \cdot \frac{1}{n}} = \lim_{\infty} \frac{2 - \frac{1}{n}}{1 + \frac{4}{n}} \xrightarrow{0} = \frac{2}{1} = 2$$

$$d) \lim_{n \rightarrow \infty} \left(\frac{2n+5}{2n}\right)^{3n-7} = \lim_{\infty} \left(\frac{2n}{2n} + \frac{5}{2n}\right)^{3n-7} = \lim_{\infty} \left[\left(1 + \frac{1}{\frac{2n}{5}}\right)^{\frac{2n}{5}}\right]^{\frac{3n-7}{\frac{2n}{5}}} = e^{\frac{15}{2}}$$

$$\lim_{\infty} \frac{\frac{3n-7}{2n}}{\frac{2n}{5}} = \lim_{\infty} \frac{\frac{3n-7}{1}}{\frac{2n}{5}} = \lim_{\infty} \frac{5 \cdot (3n-7)}{2n} = \lim_{\infty} \frac{15n-35}{2n} \cdot \frac{1}{n} = \lim_{\infty} \frac{15 - \frac{35}{n}}{2} \xrightarrow{0} = \frac{15}{2}$$

$$e) \lim_{n \rightarrow \infty} \left(\frac{2n+1}{2n-3}\right)^{3n} = \lim_{n \rightarrow \infty} \left(\frac{2n-3+4}{2n-3}\right)^{3n} = \lim_{\infty} \left(\frac{2n-3}{2n-3} + \frac{4}{2n-3}\right)^{3n} = \lim_{\infty} \left[\left(1 + \frac{1}{\frac{2n-3}{4}}\right)^{\frac{2n-3}{4}}\right]^{\frac{3n}{\frac{2n-3}{4}}} = e^6$$

$$\lim_{n \rightarrow \infty} \frac{\frac{3n}{2n-3}}{\frac{4}{2n-3}} = \lim_{\infty} \frac{\frac{12n}{2n-3} \cdot \frac{1}{n}}{\frac{4}{2n-3}} = \lim_{\infty} \frac{12}{2 - \frac{3}{n}} \xrightarrow{0} = \frac{12}{2} = 6$$

(2e2o4) f)  $\lim_{n \rightarrow \infty} \left( \frac{n-1}{n+3} \right)^{5-4n} = \lim_{\infty} \left( \frac{n+3-4}{n+3} \right)^{5-4n} = \lim_{\infty} \left( \frac{\frac{n+3}{n+3} + \frac{-4}{n+3}}{n+3} \right)^{5-4n} = \lim_{\infty} \left[ \left( 1 + \frac{1}{\frac{n+3}{-4}} \right)^{\frac{n+3}{-4}} \right]^{\frac{5-4n}{\frac{n+3}{-4}}} \stackrel{\textcircled{*}}{=} e^{16}$

(19)

$$\textcircled{*} \lim_{n \rightarrow \infty} \frac{\frac{5-4n}{n+3}}{-4} = \lim_{\infty} \frac{-4 \cdot (5-4n)}{n+3} = \lim_{\infty} \frac{-20+16n}{n+3} \cdot \frac{1}{n} = \lim_{\infty} \frac{16 - \frac{20}{n} \xrightarrow{0}}{1 + \frac{3}{n}} = \frac{16}{1} = \boxed{16}$$

g)  $\lim_{n \rightarrow \infty} \left( \frac{6+4n}{2+4n} \right)^{3-2n} = \lim_{n \rightarrow \infty} \left( \frac{2+4n+4}{2+4n} \right)^{3-2n} = \lim_{\infty} \left( \frac{2+4n}{2+4n} + \frac{4}{2+4n} \right)^{3-2n} = \lim_{\infty} \left[ \left( 1 + \frac{1}{\frac{2+4n}{4}} \right)^{\frac{2+4n}{4}} \right]^{\frac{3-2n}{\frac{2+4n}{4}}} \stackrel{\textcircled{*}}{=} e^{-2}$

$$\textcircled{*} \lim_{n \rightarrow \infty} \frac{\frac{3-2n}{2+4n}}{\frac{4}{4}} = \lim_{\infty} \frac{4 \cdot (3-2n)}{2+4n} = \lim_{n \rightarrow \infty} \frac{12-8n}{2+4n} \cdot \frac{1}{n} = \lim_{\infty} \frac{\frac{12}{n}-8 \xrightarrow{0}}{\frac{2}{n}+4} = \frac{-8}{4} = \boxed{-2}$$

h)  $\lim_{n \rightarrow \infty} \left( \frac{7n+10}{1+7n} \right)^{\frac{n}{3}} = \lim_{\infty} \left( \frac{1+7n+9}{1+7n} \right)^{\frac{n}{3}} = \lim_{\infty} \left( \frac{1+7n}{1+7n} + \frac{9}{1+7n} \right)^{\frac{n}{3}} = \lim_{\infty} \left[ \left( 1 + \frac{1}{\frac{1+7n}{9}} \right)^{\frac{1+7n}{9}} \right]^{\frac{\frac{n}{3}}{\frac{1+7n}{9}}} \stackrel{\textcircled{*}}{=} e^{\frac{3}{7}}$

$$\textcircled{*} \lim_{n \rightarrow \infty} \frac{\frac{n}{3}}{\frac{1+7n}{9}} = \lim_{\infty} \frac{\frac{9n}{3 \cdot (1+7n)}}{1+7n} = \lim_{n \rightarrow \infty} \frac{3n}{1+7n} \cdot \frac{1}{n} = \lim_{\infty} \frac{3}{\frac{1}{n}+7} \xrightarrow{0} = \frac{3}{7}$$

i)  $\lim_{n \rightarrow \infty} \left( \frac{3n+6}{3n-1} \right)^{n^2} = \lim_{\infty} \left( \frac{3n-1+7}{3n-1} \right)^{n^2} = \lim_{\infty} \left( \frac{3n-1}{3n-1} + \frac{7}{3n-1} \right)^{n^2} = \lim_{\infty} \left[ \left( 1 + \frac{1}{\frac{3n-1}{7}} \right)^{\frac{3n-1}{7}} \right]^{\frac{n^2}{\frac{3n-1}{7}}} \stackrel{\textcircled{*}}{=} e^{\infty} = \infty$

$$\textcircled{*} \lim_{n \rightarrow \infty} \frac{\frac{n^2}{3n-1}}{\frac{7}{7}} = \lim_{\infty} \frac{7n^2 \cdot \frac{1}{n}}{3n-1} = \lim_{\infty} \frac{7n \xrightarrow{0}}{3 - \frac{1}{n}} = \frac{\infty}{3} = \boxed{\infty}$$

(2.2.4)

$$j) \lim_{n \rightarrow \infty} \left( \frac{2n-5}{2n-2} \right)^{4n^2} = \lim_{\infty} \left( \frac{2n-2-3}{2n-2} \right)^{4n^2} = \lim_{\infty} \left( \frac{2n-2}{2n-2} + \frac{-3}{2n-2} \right)^{4n^2} = \lim_{\infty} \left[ \left( 1 + \frac{1}{\frac{2n-2}{-3}} \right)^{\frac{2n-2}{-3}} \right]^{\frac{4n^2}{2n-2}} \stackrel{\textcircled{20}}{=} e^{-\infty} = 0$$

$$\textcircled{*} \lim_{n \rightarrow \infty} \frac{\frac{4n^2}{2n-2}}{-3} = \lim_{n \rightarrow \infty} \frac{-12n^2 \cdot \frac{1}{n}}{2n-2} = \lim_{\infty} \frac{-12n}{2 - \frac{2}{n}} \stackrel{\textcircled{20}}{\rightarrow} \infty = \frac{-\infty}{2} = -\infty$$

$$k) \lim_{\infty} \left[ \ln \left( \frac{3n+1}{3n-5} \right)^{2n} \right] = \lim_{n \rightarrow \infty} \left[ (2-n) \cdot \ln \left( \frac{3n-5+6}{3n-5} \right) \right] = \lim_{\infty} \left[ (2-n) \cdot \ln \left( \frac{3n-5}{3n-5} + \frac{6}{3n-5} \right) \right] = \lim_{n \rightarrow \infty} \left( (2-n) \cdot \ln \left( 1 + \frac{1}{\frac{3n-5}{6}} \right)^{\frac{3n-5}{6}} \right) \\ = \lim_{n \rightarrow \infty} \left[ (2-n) \cdot \frac{1}{\frac{3n-5}{6}} \cdot \ln \left( 1 + \frac{1}{\frac{3n-5}{6}} \right)^{\frac{3n-5}{6}} \right] = \lim_{\infty} \underbrace{\left[ \frac{6 \cdot (2-n)}{3n-5} \cdot \ln \left( 1 + \frac{1}{\frac{3n-5}{6}} \right)^{\frac{3n-5}{6}} \right]}_{\rightarrow e} \stackrel{\textcircled{20}}{=} -2 \cdot \ln e = -2 \cdot 1 = -2$$

$$\textcircled{*} \lim_{n \rightarrow \infty} \frac{6 \cdot (2-n)}{3n-5} = \lim_{\infty} \frac{\frac{12-6n}{3n-5} \cdot \frac{1}{n}}{\frac{1}{n}} = \lim_{\infty} \frac{\frac{12}{n}-6}{3-\frac{5}{n}} \stackrel{\textcircled{20}}{\rightarrow} 0 = -\frac{6}{3} = -2$$

|  |
|--|
| $\text{Plati: } \ln e = 1$<br>$\log_e e = 1$ |
|--|

Vo výpočte som 2x využil vzťah:  $\log(x^y) = y \cdot \log x$

(2.2.4)

$$l) \lim_{n \rightarrow \infty} (n+1) \cdot [\ln(n+1) - \ln(n+2)] = \lim_{n \rightarrow \infty} [(n+1) \cdot \ln \frac{n+1}{n+2}] = \lim_{n \rightarrow \infty} [(n+1) \cdot \ln \left( \frac{n+2}{n+2} + \frac{-1}{n+2} \right)] =$$

Platí:  $\log x + \log y = \log(x \cdot y)$   
 $\log x - \log y = \log \left( \frac{x}{y} \right)$

$$= \lim_{n \rightarrow \infty} \left( (n+1) \cdot \ln \left[ \left( 1 + \frac{1}{\frac{n+2}{-1}} \right)^{\frac{n+2}{-1}} \right]^{\frac{1}{\frac{n+2}{-1}}} \right) = \lim_{n \rightarrow \infty} \left( \frac{-(n+1)}{n+2} \cdot \ln \underbrace{\left( 1 + \frac{1}{\frac{n+2}{-1}} \right)^{\frac{n+2}{-1}}}_{\rightarrow e} \right) = -1 \cdot \ln e = -1 \cdot 1 = -1$$

$$\textcircled{*} \lim_{n \rightarrow \infty} \frac{-(n+1)}{n+2} = \lim_{n \rightarrow \infty} \frac{-n-1}{n+2} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{-1 - \left( \frac{1}{n} \right)^0}{1 + \left( \frac{2}{n} \right)} = \frac{-1}{1} = -1$$

(3.1.1)

$$a) \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 2x}$$

ak máme limitu funkcie a  $x$  sa blíži k nejakému číslu (v tomto prípade  $x \rightarrow 2$ ), tak vždy skusime najprv číslo dosadiť za  $x$ :

$$\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 2x} \stackrel{?}{=} \frac{2^2 - 2 - 2}{2^2 - 2 \cdot 2} = \frac{0}{0} \rightarrow \text{nemá zmysel}$$

~ výslo nám  $\frac{0}{0}$ , čo je nesmysel.

Postupujeme teda tak, že čitatelia aj menovatelia upravíme na

Evar:  $\frac{(x-a) \cdot \text{necō}}{(x-a) \cdot \text{necō}}$ , kde  $\textcircled{a}$  je číslo,

ke ktorému sa blíži  $x$  (u nás  $a=2$ ):

[ keby po dosadení výslo niečo pekné, teda nejaké číslo alebo  $\pm \infty$ , bol by to hned výsledok ]

(21)

$$\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 2x} = \lim_{x \rightarrow 2} \frac{(x-2)(x+1)}{x \cdot (x-2)} = \lim_{\substack{x \rightarrow 2 \\ \text{dosačitme}}} \frac{x+1}{x} = \frac{2+1}{2} = \underline{\underline{\frac{3}{2}}}$$

$$x^2 - x - 2: D = 1 + 4 \cdot 2 = 9$$

$$x_{1,2} = \frac{-(-1) \pm \sqrt{9}}{2} = \begin{cases} 2 \\ -1 \end{cases} \Rightarrow x^2 - x - 2 = (x-2)(x+1)$$

(30101)

b)  $\lim_{x \rightarrow 0} \frac{x^3 - 4x}{2x^2 + 3x} = \lim_{x \rightarrow 0} \frac{x \cdot (x^2 - 4)}{x \cdot (2x+3)} = \lim_{x \rightarrow 0} \frac{x^2 - 4}{2x+3} = \frac{0^2 - 4}{2 \cdot 0 + 3} = \underline{\underline{\frac{-4}{3}}}$

c)  $\lim_{x \rightarrow -1} \frac{x^3}{(x+1)^2} = \frac{(-1)^3}{(-1+1)^2} = \frac{-1}{0} \Rightarrow \underline{\underline{\text{neexistuje}}}$

ak výjde  $\frac{\pm\infty}{\pm\infty}$ ,  $\frac{\pm\infty}{0}$ ,  $\frac{0}{\pm\infty}$ ,  $\frac{0}{0}$  → musíme rátať inak

ak výjde  $\frac{\text{cislo}^{\neq 0}}{0}$  ⇒  $\underline{\underline{\text{limita}}}$

d)  $\lim_{x \rightarrow -1} \frac{(x+1)^2 \cdot (x-1)}{x^3 + 1} = \lim_{\substack{x \rightarrow -1 \\ \text{platí vztah: } x^3 + 1 = (x+1) \cdot (x^2 - x + 1)}} \frac{(x+1) \cdot (x+1) \cdot (x-1)}{(x+1) \cdot (x^2 - x + 1)} = \frac{(1+1) \cdot (-1-1)}{(-1)^2 - (-1) + 1} = \frac{0}{3} = \underline{\underline{0}}$

platí vztah:  $x^3 + 1 = (x+1) \cdot (x^2 - x + 1)$

3.102

a)  $f: y = \begin{cases} \frac{x^2-9}{x+3} & x \neq -3 \\ 6 & x = -3 \end{cases}$  → je to zložené zapísaná funkcia a znamená:  
 ak  $x = -3$  tak  $y = 6$   
 inak  $y = \frac{x^2-9}{x+3}$

→ ci je funkcia spojite zistime tak, že nájdeme limitu  $\frac{x^2-9}{x+3}$  pre  $x \rightarrow -3$   
 a, zistime, ci tato limita = 6. Ak áno, funkcia je spojite. Inak nie je.

tu je chyba v zadani, má tam byť  $x = -3$

$$\lim_{x \rightarrow -3} \frac{x^2-9}{x+3} = \lim_{x \rightarrow -3} \frac{(x+3)(x-3)}{x+3} = \lim_{x \rightarrow -3} (x-3) = -3-3 = -6 \neq 6 \Rightarrow \text{funkcia nie je spojite}$$

$a^2 - b^2 = (a+b)(a-b)$

b)  $f: y = \begin{cases} \frac{x^3-4x}{x-2} & x \neq 2 \\ 8 & x = 2 \end{cases}$

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{x^3-4x}{x-2} &= \lim_{x \rightarrow 2} \frac{x(x^2-4)}{x-2} = \lim_{x \rightarrow 2} \frac{x(x-2)(x+2)}{x-2} = \lim_{x \rightarrow 2} \frac{x(x+2)}{1} = \\ &= 2 \cdot (2+2) = 8 \Rightarrow \text{funkcia je spojite} \end{aligned}$$

c)  $f: y = \begin{cases} 3+x^2 & x \leq 0 \\ \frac{\sin 3x}{x} & x > 0 \end{cases}$

dokazatý vzorec:  
 $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

→ zistime, ci  $\lim_{x \rightarrow 0} (3+x^2) = \lim_{x \rightarrow 0} \frac{\sin 3x}{x}$

$$\lim_{x \rightarrow 0} (3+x^2) = 3+0^2 = 3$$

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{x} = \lim_{x \rightarrow 0} \frac{3 \cdot \sin 3x}{3x} = 3 \cdot 1 = 3$$

limity sa rovnajú  
 ⇒ funkcia je spojite

(24)

3.103

a)  $f: y = \begin{cases} ax & x < 1 \\ 2 - \frac{x}{a} & x \geq 1 \end{cases}$

musí platit:  $\lim_{x \rightarrow 1^-} ax = \lim_{x \rightarrow 1^+} (2 - \frac{x}{a})$

$$\lim_{x \rightarrow 1^-} ax = a \cdot 1 = a , \quad \lim_{x \rightarrow 1^+} (2 - \frac{x}{a}) = 2 - \frac{1}{a}$$

$$a = 2 - \frac{1}{a} / \cdot a$$

$$a^2 - 2a + 1 = 0$$

$$(a-1)^2 = 0 \Rightarrow \underline{\underline{a=1}}$$

b)  $f: y = \begin{cases} e^{ax} & x < 0 \\ a-x & x \geq 0 \end{cases}$

$$\lim_{x \rightarrow 0} e^{ax} = e^{a \cdot 0} = e^0 = 1 \quad \left. \right\} a=1$$

$$\lim_{x \rightarrow 0} (a-x) = a-0 = a \quad \left. \right\} a=1$$

c)  $f: y = \begin{cases} \frac{-1}{x^2+a} & x < 0 \\ (\frac{1}{3})^x & x \geq 0 \end{cases}$

$$\lim_{x \rightarrow 0} \frac{-1}{x^2+a} = \frac{-1}{0+a} = -\frac{1}{a} \quad \left. \right\} -\frac{1}{a} = 1/a$$

$$\lim_{x \rightarrow 0} (\frac{1}{3})^x = (\frac{1}{3})^0 = 1 \quad \left. \right\} -1 = a \rightsquigarrow \underline{\underline{a=-1}}$$

d)  $f: y = \begin{cases} e^{-\frac{1}{x}} & x \leq -1 \\ x^2+ax & x > -1 \end{cases}$

$$\lim_{x \rightarrow -1} e^{-\frac{1}{x}} = e^{-\frac{1}{-1}} = e^1 = e \quad \left. \right\} e = 1-a$$

$$\lim_{x \rightarrow -1} (x^2+ax) = (-1)^2 + a \cdot (-1) = 1 - a \quad \left. \right\} a = 1 - e$$

3.1.4

$$a) f: y = \frac{x}{x+4}$$

Máme 2 druhé asymptoty:

1. Asymptoty bez smernice:

$$\lim_{x \rightarrow a^{\pm}} f(x) = \pm\infty \Rightarrow \boxed{x=a} \text{ je as. bez sm. (ABS)}$$

→ limita sprava/zľava

2. Asymptoty so smernicou:

[existujúce pre  $x \rightarrow \infty$  a pre  $x \rightarrow -\infty$ ]

$$\text{ak } p = \lim_{x \rightarrow \infty} \frac{f(x)}{x} \text{ a } q = \lim_{x \rightarrow \infty} [f(x) - p \cdot x]$$

$$\Rightarrow \boxed{y = px + q} \text{ je as. so sm. (ASS)}$$

ABS:

$$\lim_{x \rightarrow -4^-} \frac{x}{x+4} = -\infty \Rightarrow \boxed{x = -4} \text{ je ABS}$$

$$\lim_{x \rightarrow -4^+} \frac{x}{x+4} = \infty \Rightarrow \text{stačí zistiť jednu jednostrannú limitu (sprava alebo zľava) či je } \pm\infty$$

↓  
 x sa blíži bodek nespojitosťi danej funkcie (zľava alebo sprava), t.j. bodu, pre ktorý nie je daná funkcia definovaná (v tomto prípade k -4, pretože  $-4+4=0$  a to v menovateli nesmie byť)  
 ~ limita sa rovná  $\infty$  - zistíme vším jedno druhu tak, že za x dosadíme číslo, ktoré sa blíži k (-4) zľava, teda napr.  $-3,999999999 \Rightarrow$  výjde obrovské záporné číslo a teda  
 určite limita pôjde do  $-\infty$  (neformálne zistovanie hodnoty limity):  $\frac{-3,999...}{-3,999...+4} = \underline{-\infty}$

ASS:

$$\lim_{x \rightarrow \infty} \frac{\frac{x}{x+4}}{x} = \lim_{x \rightarrow \infty} \frac{x}{x \cdot (x+4)} = \lim_{x \rightarrow \infty} \frac{1}{x+4} = 0 = p$$

$$q = \lim_{x \rightarrow \infty} \left( \frac{x}{x+4} - 0 \cdot x \right) = \lim_{x \rightarrow \infty} \frac{x}{x+4} \cdot \frac{1}{x} = \lim_{x \rightarrow \infty} \frac{1}{1 + \frac{4}{x}} = \boxed{1 = q}$$

$y = 1$  je ASS (pre  $\lim_{x \rightarrow \infty}$  výjdu  $0 \cdot x + 1$ )  
 Nav q rovnaké)

(26)

3.10.4

b)  $f: y = \frac{1-x^2}{x-2}$

ABS:  $\lim_{x \rightarrow 2^-} \frac{1-x^2}{x-2} = \frac{1-4}{1,999\dots - 2} = \frac{-3}{-0,00\dots 01} = \infty \Rightarrow x=2 \text{ je ABS}$

~~(je skončená funkcia)~~

toto si treba písat niekde naspäť  
a zapísanie už len  $\lim = \infty$

ASS:  $\lim_{x \rightarrow \infty} \frac{\frac{1-x^2}{x-2}}{x} = \lim_{\infty} \frac{1-x^2}{x(x-2)} = \lim_{\infty} \frac{1-x^2}{x^2-2x} \cdot \frac{1}{x^2} = \lim_{\infty} \frac{\frac{1}{x^2}-1}{1-\frac{2}{x}} = \frac{\frac{1}{1^2}-1}{1-\frac{2}{1}} = \frac{-1}{1} = -1$

$$\begin{aligned} \lim_{\infty} \left( \frac{1-x^2}{x-2} + 1 \cdot x \right) &= \lim_{\infty} \left( \frac{1-x^2}{x-2} + x \right) = \lim_{\infty} \left( \frac{1-x^2+x \cdot (x-2)}{x-2} \right) = \lim_{\infty} \frac{1-x^2+x^2-2x}{x-2} = \\ &= \lim_{\infty} \frac{1-2x}{x-2} \cdot \frac{1}{x} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}-2}{1-\frac{2}{x}} = \frac{-2}{1} = -2 \end{aligned} \Rightarrow y = -x-2 \text{ je ASS}$$

c)  $f: y = \frac{2x^2}{2x-1}$

ABS:  $\lim_{x \rightarrow \frac{1}{2}^+} \frac{2x^2}{2x-1} = \frac{\frac{1}{2}}{0,00\dots 01} = \infty \Rightarrow x=\frac{1}{2} \text{ je ABS}$

ASS:  $\lim_{x \rightarrow \infty} \frac{\frac{2x^2}{2x-1}}{x} = \lim_{\infty} \frac{2x}{x(2x-1)} = \lim_{\infty} \frac{2x}{2x-1} \cdot \frac{1}{x} = \lim_{\infty} \frac{2}{2-\frac{1}{x}} = 1$

$$\begin{aligned} \lim_{\infty} \left( \frac{2x^2}{2x-1} - 1 \cdot x \right) &= \lim_{\infty} \frac{2x^2-x(2x-1)}{2x-1} = \lim_{\infty} \frac{2x^2-2x^2+x}{2x-1} = \lim_{\infty} \frac{x}{2x-1} = \\ &= \lim_{\infty} \frac{1}{2-\frac{1}{x}} = \frac{1}{2} \end{aligned} \Rightarrow y = x + \frac{1}{2} \text{ je ASS}$$

[pre  $\lim_{x \rightarrow \infty}$  rovnaké ako pre  $\lim_{x \rightarrow \infty}$ ]

3.1.4

$$d) f: y = 3x + \frac{3}{x-2} = \frac{3x(x-2)+3}{x-2} = \frac{3x^2-6x+3}{x-2} = \frac{3 \cdot (x^2-2x+1)}{x-2} = \frac{3 \cdot (x-1)^2}{x-2}$$

→ s takýmto tvárom funkcie  
sa mi bude ľahšie rútať

27

ABS:

$$\lim_{x \rightarrow 2^+} \frac{3(x-1)^2}{x-2} = \frac{3}{0,00...01} = \infty \Rightarrow \underline{x=2 \text{ je ABS}}$$

ASS:

$$\lim_{x \rightarrow \infty} \frac{\frac{3(x-1)^2}{x-2}}{x} = \lim_{x \rightarrow \infty} \frac{3(x-1)^2}{x(x-2)} = \lim_{x \rightarrow \infty} \frac{3x^2-6x+3}{x^2-2x} \cdot \frac{1}{x^2} = \lim_{x \rightarrow \infty} \frac{3 - \frac{6}{x} + \frac{3}{x^2}}{1 - \frac{2}{x}} = 3$$

$$\lim_{x \rightarrow \infty} \left( \frac{3(x-1)^2}{x-2} - 3 \cdot x \right) = \lim_{x \rightarrow \infty} \frac{3(x-1)^2 - 3x(x-2)}{x-2} = \lim_{x \rightarrow \infty} \frac{3x^2-6x+3 - 3x^2+6x}{x-2} = \lim_{x \rightarrow \infty} \frac{3}{x-2} = 0$$

$\Rightarrow \underline{y = 3x \text{ je ASS [pre } \lim_{x \rightarrow \infty} \text{ rovnaké]}}$

$$f: y = x \cdot e^{\frac{1}{x^2}}$$

$$\text{ABS: } \lim_{x \rightarrow 0^+} x \cdot e^{\frac{1}{x^2}} = \lim_{x \rightarrow 0^+} \frac{1}{\frac{1}{x^2}} \cdot e^{\frac{1}{x^2}} = \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{\frac{1}{x^2}} \cdot e^{\frac{1}{x^2}} = \lim_{x \rightarrow 0^+} \frac{e^{\frac{1}{x^2}}}{\frac{1}{x^2}} \cdot \frac{1}{x} = \lim_{x \rightarrow 0^+} \frac{e^{\frac{1}{x^2}}}{\frac{1}{x^2}} \cdot \frac{1}{x} = \infty \cdot \infty = \infty \Rightarrow \underline{x=0 \text{ je ABS}}$$

platí:  $\lim_{x \rightarrow 0} \frac{1}{x} \nexists$

$$\lim_{x \rightarrow 0^+} \frac{1}{x} = \infty$$

$$\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$$

$\lim_{x \rightarrow \infty} \frac{a^x}{x} = \infty \text{ (ak } a > 1)$

$\lim_{x \rightarrow \infty} \frac{x}{a^x} = 0 \text{ (ak } a > 1)$

( $e = 2,71828\dots$ )

$0^+$  je skratený zápis pre  
„nula sprava“, t. j.  
pre  $\lim_{x \rightarrow 0^+} x$

3.1.4

(28)

f) ASS:  $\lim_{\infty} \frac{xe^{\frac{1}{x^2}}}{x} = \lim_{\infty} e^{\left(\frac{1}{x^2}\right)^0} = e^0 = 1$

$$\lim_{\infty} (xe^{\frac{1}{x^2}} - 1 \cdot x) = \lim_{\infty} x \cdot (e^{\frac{1}{x^2}} - 1) = \lim_{\infty} \frac{e^{\frac{1}{x^2}} - 1}{\frac{1}{x^2}} \cdot \left(\frac{1}{x}\right)^0 = 1 \cdot 0 = 0$$

$y=x$  je ASS  
[pre  $\lim_{x \rightarrow \infty}$  rovnaké]

dôležitý výrok:  $\boxed{\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1}$

my máme  $\lim_{x \rightarrow \infty} \frac{e^{\frac{1}{x^2}} - 1}{\frac{1}{x^2}}$  a platí:  $\lim_{x \rightarrow \infty} \frac{1}{x^2} = 0$

~~(28)~~  
g)

ABS: za  $x$  môžeme dosať ľakšie ( $D(f) = (-\infty; \infty)$ )  $\Rightarrow$  ABS

ASS:  $\lim_{x \rightarrow \infty} \frac{4xe^{-x^2}}{x} = \lim_{\infty} 4e^{-x^2} = 4 \cdot e^{-\infty} = \frac{4}{e^{\infty}} = \frac{4}{\infty} = 0$  [  $a^{-x} = \frac{1}{a^x}$  ]

$$\lim_{x \rightarrow \infty} (4xe^{-x^2} - 0) = \lim_{\infty} 4xe^{-x^2} = \lim_{\infty} \frac{4x}{e^{x^2}} = \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \frac{4x^2}{e^{x^2}} = \lim_{x \rightarrow \infty} \frac{4}{x} \cdot \frac{x^2}{e^{x^2}}$$

$= 0 \Rightarrow y=0$  je ASS [pre  $\lim_{x \rightarrow \infty}$  rovnaké]

lebo  $\lim_{\infty} \frac{x}{a^x} = 0$  pre  $a > 1$

h)  $f: y = x + e^{-x}$

ABS ? ( $D(f) = (-\infty; \infty)$ )

ASS:  $\lim_{x \rightarrow \infty} \frac{x + e^{-x}}{x} = \lim_{x \rightarrow \infty} \left( \frac{x}{x} + \frac{e^{-x}}{x} \right) = \lim_{\infty} \left( 1 + \left( \frac{1}{x \cdot e^x} \right)^0 \right) = 1$

$$\lim_{\infty} (x + e^{-x} - 1 \cdot x) = \lim_{\infty} e^{-x} = \lim_{\infty} \frac{1}{e^x} = 0$$

$y=x$  je ASS

(3.104) h)  $\lim_{x \rightarrow -\infty} \frac{x + e^x}{x} = \lim_{x \rightarrow -\infty} \left(1 + \frac{e^x}{x}\right) = \lim_{x \rightarrow -\infty} \left(1 - \frac{e^{-x}}{-x}\right) = 1 - \frac{e^\infty}{\infty} = 1 - \infty = -\infty \Rightarrow$  pre tento prípad  $p = -\infty$

(29)

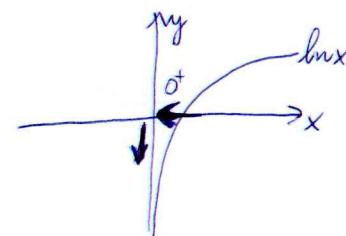
~ pokiaľ nám výsledok  $p = \pm\infty$ , v danom prípade ASS  $\nexists$

~  $\nexists$  teda iba jedna ASS:  $y = x$  (pre  $x \rightarrow \infty$ )

~ na skúšku a na zároveň odporučam vyrátať ASS pre  $x \rightarrow \infty$  aj pre  $x \rightarrow -\infty$ , pretože sa môže stať, že pre každý prípad výsledok iná asymptota

i)  $f: y = x + \frac{\ln x}{x}$  ABS:  $D(f) = (0, \infty)$ , lebo  $x \neq 0 \wedge x > 0$

$$\lim_{x \rightarrow 0^+} \left(x + \frac{\ln x}{x}\right) = \lim_{x \rightarrow 0^+} \left(x + \frac{1/x \cdot \ln x}{1}\right) = \lim_{x \rightarrow 0^+} \left(x + \frac{\ln x}{x}\right) = \lim_{x \rightarrow 0^+} x = \underline{-\infty} \Rightarrow x = 0 \text{ je ABS}$$



ak si pozrieme graf  $\ln x$ , vidime, že pre  $x \rightarrow 0^+$  sa blíži graf do  $-\infty$

ASS:  $\lim_{x \rightarrow \infty} \frac{x + \frac{\ln x}{x}}{x} = \lim_{x \rightarrow \infty} \frac{x + \frac{x^2 + \ln x}{x}}{x} = \lim_{x \rightarrow \infty} \frac{x + \frac{x^2}{x} + \frac{\ln x}{x}}{x} = \lim_{x \rightarrow \infty} \left(1 + \frac{\ln x}{x^2}\right) = \underline{1}$

[pre  $\lim_{x \rightarrow \infty} p \nexists$ , lebo  $\lim_{x \rightarrow \infty} (\ln x) \nexists$ ]

platí:  $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0$

$$\lim_{x \rightarrow \infty} \left(x + \frac{\ln x}{x} - 1 \cdot x\right) = \lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0 \Rightarrow y = x \text{ je ASS}$$

(4.1)

a)  $f: y = x^2 - 2x + 2 ; T \in [0; ?]$

~ najprv zistíme y-ovú súradnicu bodu T:

$$y = 0^2 - 2 \cdot 0 + 2 = 2 \quad (\text{dosadili sme } x\text{-ovú súradnicu do predpisu funkcie})$$

$T \in [0; 2]$

~ potom zistíme deriváciu funkcie:

$$f' = y' = (x^2 - 2x + 2)' = 2x - 2,$$

~ nакoniec dosadíme do vzorcov:

dotyčnica:  $y = 2 + (2 \cdot 0 - 2) \cdot (x - 0) = 2 + (-2) \cdot x = 2 - 2x = -2x + 2$

$$\underline{\underline{y = -2x + 2}}$$

normála:  $y = 2 + \frac{-1}{2 \cdot 0 - 2} \cdot (x - 0) = 2 + \frac{-1}{-2} \cdot x = 2 + \frac{1}{2}x$

$$\underline{\underline{y = \frac{1}{2}x + 2}}$$

b)  $f: y = 2x - x^2 ; T = [1; ?]$

$$y = 2 \cdot 1 - 1^2 = 1 \Rightarrow \underline{\underline{T[1; 1]}}$$

$$f' = y' = 2 - 2x$$

(30)

Vzorce:

~ rovnica dotyčnice:  $y = f(a) + f'(a) \cdot (x-a)$   
 v bode  $A[a, f(a)]$

~ rovnica normály:  $y = f(a) + \frac{-1}{f'(a)} \cdot (x-a)$   
 v bode  $A[a, f(a)]$

$\nearrow$  ovač súr. T  $\nearrow$  v x-ovej súr. T

dotyčnica:  $y = 2 + (2 \cdot 1 - 2) \cdot (x - 1) = 2 + 0 \cdot (x - 1) = 2$

$$\underline{\underline{y = 2}}$$

normála:  $y = 2 + \frac{-1}{2 \cdot 1 - 2} \cdot (x - 1) = 2 + \frac{-1}{0} \cdot (x - 1) = 2 - \frac{1}{0} \cdot x = 2 + \frac{1}{2}x$

$$\underline{\underline{y = \frac{1}{2}x + 2}}$$

dotyčnica:  $y = 1 + (2 - 2 \cdot 1) \cdot (x - 1) = 1 + 0 \cdot (x - 1) = 1$

$$\underline{\underline{y = 1}}$$

normála:  $y = 1 + \frac{-1}{2 - 2 \cdot 1} \cdot (x - 1) = 1 - \frac{1}{0} \cdot (x - 1) \rightsquigarrow \frac{1}{0} \text{ nemá zmysel}$

normála  $\exists$

4.1

c)  $f: y = 1 - \frac{1}{x+1} ; T = [0; ?]$

$$y(0) = 1 - \frac{1}{0+1} = 0 \Rightarrow T[0; 0]$$

$$y' = -\frac{-(x+1)^2}{(x+1)^2} = \underbrace{\frac{+1}{(x+1)^2}}$$

dotyčnica:

$$y = 0 + \frac{1}{(0+1)^2} \cdot (x-0) = \frac{1}{1} \cdot x$$

$$\underline{\underline{y=x}}$$

normála:

$$y = 0 + \frac{-1}{\frac{1}{(0+1)^2}} \cdot (x-0) = -\frac{1}{1} \cdot (x-0) = -x$$

$$\underline{\underline{y=-x}}$$

d)  $f: y = \sqrt[3]{x+4} ; T = [-3; ?]$

$$y(-3) = \sqrt[3]{-3+4} = 1 \Rightarrow T[-3; 1]$$

$$y' = [(x+4)^{\frac{1}{3}}]' = \frac{1}{3}(x+4)^{-\frac{2}{3}} \cdot (x+4)' = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{(x+4)^2}} \cdot 1 = \underbrace{\frac{1}{3 \cdot \sqrt[3]{(x+4)^2}}}$$

dotyčnica:

$$y = 1 + \frac{1}{3 \cdot \sqrt[3]{(-3+4)^2}} \cdot (x+3) = 1 + \frac{1}{3} \cdot (x+3) \\ = 1 + \frac{1}{3}x + 1$$

$$\underline{\underline{y = \frac{1}{3}x + 2}}$$

e)  $f: y = \frac{2x}{x+1} ; T[0; ?]$

$$y(0) = \frac{2 \cdot 0}{0+1} = 0 \Rightarrow T[0; 0]$$

$$f' = \frac{2 \cdot (x+1) - 2x \cdot 1}{(x+1)^2} = \frac{2x+2-2x}{(x+1)^2} = \underbrace{\frac{2}{(x+1)^2}}$$

$$\text{normála: } y = 1 + \frac{-1}{\frac{2}{(0+1)^2}} \cdot (x+3) = 1 - 3 \cdot (x+3) \\ = 1 - 3x - 9$$

$$\underline{\underline{y = -3x - 8}}$$

4.1 e) dotyčnica:  $y = 0 + \frac{2}{(0+1)^2} \cdot (x-0) = \frac{2}{1} \cdot x \Rightarrow \underline{\underline{y=2x}}$

normála:  $y = 0 + \frac{-1}{\frac{2}{1}} \cdot (x-0) = -\frac{1}{2}x \Rightarrow \underline{\underline{y=-\frac{1}{2}x}}$

f)  $f: y = x + \sqrt{4-x}$ ;  $T = [3; ?]$

$$y(3) = 3 + \sqrt{4-3} = 4 \Rightarrow T[3; 4]$$

$$y = 1 + [(4-x)^{\frac{1}{2}}] = 1 + \frac{1}{2}(4-x)^{-\frac{1}{2}} \cdot (-1) = 1 + \frac{1}{2} \cdot \frac{1}{\sqrt{4-x}} \cdot (-1) = 1 + \underbrace{\frac{1}{2 \cdot \sqrt{4-x}}}_{\text{dotyčnica}}$$

$$\begin{aligned} \text{dotyčnica: } y &= 4 + 1 - \frac{1}{2 \cdot \sqrt{4-3}} \cdot (x-3) = \\ &= 5 - \frac{1}{2} \cdot (x-3) = 5 - \frac{1}{2}x + \frac{3}{2} \end{aligned}$$

$$\underline{\underline{y = -\frac{1}{2}x + \frac{13}{2}}}$$

g)  $f: y = (x-1) \cdot e^x$ ;  $T = [1; ?]$

$$y(1) = 0 \cdot e = 0 \Rightarrow T = [1; 0]$$

$$f = 1 \cdot e^x + (x-1) \cdot e^x = e^x \cdot (1+x-1) = \underline{\underline{x \cdot e^x}}$$

dotyčnica:  $y = 0 + 1 \cdot e^1 \cdot (x-1) = e \cdot (x-1) = ex - e$

$$\underline{\underline{y = ex - e}}$$

normála:  $y = 0 + \frac{-1}{e} \cdot (x-1) = -\frac{1}{e}x + \frac{1}{e}$

$$\underline{\underline{y = -\frac{1}{e}x + \frac{1}{e}}}$$

$$\underline{\underline{y = -2x + 10}}$$

(33)

(4.1)

$$h) f: y = \ln(\sin x); T = \left[\frac{\pi}{2}; ? \right]$$

$$y\left(\frac{\pi}{2}\right) = \ln(\sin \frac{\pi}{2}) = \ln 1 = 0 \Rightarrow T\left[\frac{\pi}{2}; 0\right]$$

$$y' = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \underline{\operatorname{ctg} x}$$

dotyčnica:

$$y = 0 + \operatorname{ctg} \frac{\pi}{2} \cdot \left(x - \frac{\pi}{2}\right) = 0 \cdot \left(x - \frac{\pi}{2}\right) = 0$$

$$\underline{\underline{y=0}}$$

normála:

$$y = 0 + \frac{-1}{0} \cdot \left(x - \frac{\pi}{2}\right)$$

$$\underline{\underline{\text{normála } \nexists}}$$

(4.2)

a)  $f: y = 2 \cdot \sqrt{x^2+3} ; d = 45^\circ$  platí:  $f'(x) = \operatorname{tg} d$

$$y' = 2 \cdot \left[ (x^2+3)^{\frac{1}{2}} \right]' = 2 \cdot \frac{1}{2} \cdot (x^2+3)^{-\frac{1}{2}} \cdot 2x = \\ = 2x \cdot \frac{1}{\sqrt{x^2+3}} = \frac{2x}{\sqrt{x^2+3}}$$

$$\frac{2 \cdot x_0}{\sqrt{x_0^2+3}} = \operatorname{tg} 45^\circ = 1 \quad / \cdot \sqrt{x_0^2+3}$$

$$2x_0 = \sqrt{x_0^2+3} \quad |^2$$

$$4x_0^2 = x_0^2 + 3$$

$$3x_0^2 = 3$$

$$x_0^2 = 1$$

$x_0 = \pm 1$   $\rightsquigarrow$  dosadíme do pôvodného predpisu aby sme zistili  $y_0$ :

$$y_0 = 2 \cdot \sqrt{x_0^2+3}$$

$$y_0 = 2 \cdot \sqrt{4} = 4 \Rightarrow T_1 = [-1; 4] \wedge T_2 = [1; 4]$$

dotyčnica:

$$y = 4 + \frac{2 \cdot 1}{\sqrt{1^2+3}} \cdot (x-1) = 4 + \frac{2}{2} \cdot (x-1) = 4+x-1$$

$$\underline{\underline{y=x+3}}$$

normála:

$$y = 4 + \frac{-1}{\frac{2}{2}} \cdot (x-1) = 4-x+1$$

$$\underline{\underline{y=-x+5}}$$

ak ale dosadíme  $x_0 = -1$  späť, zistíme že vztah neplatí  $\Rightarrow T = [1; 4]$

(4,2)

b)  $f: y = \operatorname{arctg} 2x ; \alpha = 45^\circ \Rightarrow \underline{\operatorname{tg} \alpha = 1}$

$$y = (\operatorname{arctg} 2x) = \frac{1}{1+(2x)^2} \cdot 2 = \frac{2}{1+4x^2}$$

$$\frac{2}{1+4x_0^2} = 1 \quad | \cdot (1+4x_0^2)$$

$$2 = 1 + 4x_0^2$$

$$4x_0^2 = 1$$

$$x_0^2 = \frac{1}{4}$$

$$x_0 = \pm \frac{1}{2} \rightarrow \text{když dosadíme späť, zistíme, že obě možnosti vyhovují}$$

zistíme  $y_0$ :

$$y_0 = \operatorname{arctg} 2 \cdot \left(\pm \frac{1}{2}\right) = \operatorname{arctg} (\pm 1)$$

$$\text{ak } x_0 = -\frac{1}{2} \Rightarrow y_0 = \operatorname{arctg} (-1) = -\frac{\pi}{4}$$

$$x_0 = \frac{1}{2} \Rightarrow y_0 = \operatorname{arctg} 1 = \frac{\pi}{4}$$

$$\left. \begin{array}{l} T_1 \left[ -\frac{1}{2}; -\frac{\pi}{4} \right] \\ T_2 \left[ \frac{1}{2}; \frac{\pi}{4} \right] \end{array} \right\}$$

1. dotyčnice:

$$y = \frac{\pi}{4} + \frac{2}{1+4 \cdot \left(\frac{1}{2}\right)^2} \cdot \left(x - \frac{1}{2}\right) = \frac{\pi}{4} + \frac{2}{2} \cdot \left(x - \frac{1}{2}\right)$$

$$\underline{\underline{y = x - \frac{1}{2} + \frac{\pi}{4}}}$$

$$1. \text{ normála: } y = \frac{\pi}{4} + \frac{-1}{\frac{2}{2}} \cdot \left(x - \frac{1}{2}\right) = \frac{\pi}{4} - x + \frac{1}{2}$$

$$\underline{\underline{y = -x + \frac{1}{2} + \frac{\pi}{4}}}$$

2. dotyčnice:

$$y = -\frac{\pi}{4} + \frac{2}{1+4 \cdot \left(-\frac{1}{2}\right)^2} \cdot \left(x + \frac{1}{2}\right) = -\frac{\pi}{4} + \frac{2}{2} \cdot \left(x + \frac{1}{2}\right)$$

$$\underline{\underline{y = x + \frac{1}{2} - \frac{\pi}{4}}}$$

2. normála:

$$y = -\frac{\pi}{4} + \frac{-1}{\frac{2}{2}} \cdot \left(x + \frac{1}{2}\right) = -\frac{\pi}{4} - x - \frac{1}{2}$$

$$\underline{\underline{y = -x - \frac{1}{2} - \frac{\pi}{4}}}$$

~~(konec)~~

(4.3)

a)  $f: y = \ln(x+1)$ ;  $p: y = x+2$ ;  $A \parallel p \rightarrow$  keďže sú dané priamky rovnobežné, majú rovnaké smernice, t.j. číslo pred „ $x$ “ v danom predpise  $y = x+2 = 1 \cdot x + 2$  a teda smernica dotyčnice bude tiež  $1$

$$y = \frac{1}{x+1}$$

$$\frac{1}{x_0+1} = 1$$

$$1 = x_0 + 1$$

$$x_0 = 0$$

$$y_0 = \ln(x_0 + 1) = \ln(0 + 1) = 0 \Rightarrow T[0;0]$$

dotyčnica:

$$y = 0 + \frac{1}{0+1} \cdot (x-0) = x \Rightarrow \underline{\underline{y = x}}$$

normála:

$$y = 0 + \frac{-1}{\frac{1}{1}} \cdot (x-0) \Rightarrow \underline{\underline{y = -x}}$$

b)  $f: y = 3 - 2 \cdot e^{\frac{x}{2}}$ ;  $p: 2x + 2y - 3 = 0$ ;  $p \parallel A$

$$2y = -2x + 3$$

$$y = -x + \frac{3}{2}$$

$$y = -2 \cdot e^{\frac{x}{2}} \cdot \frac{1}{2} = -e^{\frac{x}{2}}$$

$$-e^{\frac{x_0}{2}} = -1$$

$$e^{\frac{x_0}{2}} = 1 \quad [e^0 = 1]$$

$$\frac{x_0}{2} = 0$$

$$x_0 = 0 \rightarrow y_0 = 3 - 2 \cdot e^{\frac{0}{2}} = 1 \Rightarrow T[0;1]$$

dotyčnica:

$$y = 1 + (-e^{\frac{0}{2}}) \cdot (x-0) = 1 - 1 \cdot x \Rightarrow \underline{\underline{y = -x + 1}}$$

normála:

$$y = 1 + \frac{-1}{-1} \cdot (x-0) = 1 + x \Rightarrow \underline{\underline{y = x + 1}}$$

(4.3)

c)  $f: y = x^3 - x$ ;  $p: y = 2x$ ;  $A \parallel p$

$$\underline{y = 3x^2 - 1}$$

$$3x_0^2 - 1 = 2$$

$$3x_0^2 = 3$$

$$x_0^2 = 1$$

$$x_0 = \pm 1$$

$$\begin{aligned} x_0 = -1 &\Rightarrow y_0 = (-1)^3 - (-1) = -1 + 1 = 0 \\ x_0 = 1 &\Rightarrow y_0 = 1^3 - 1 = 0 \end{aligned} \quad \left. \begin{array}{l} T_1[-1; 0] \\ T_2[+1; 0] \end{array} \right\}$$

1. dotyčnica:

$$y = 0 + (3 \cdot (-1)^2 - 1) \cdot (x + 1) = 2 \cdot (x + 1) \Rightarrow \underline{\underline{y = 2x + 2}}$$

1. normála:

$$y = 0 + \frac{-1}{2} \cdot (x + 1) = -\frac{1}{2} \cdot (x + 1) \Rightarrow \underline{\underline{y = -\frac{1}{2}x - \frac{1}{2}}}$$

2. dotyčnica:

$$y = 0 + (3 \cdot 1^2 - 1) \cdot (x - 1) = 2 \cdot (x - 1) \Rightarrow \underline{\underline{y = 2x - 2}}$$

2. normála:

$$y = 0 + \frac{-1}{2} \cdot (x - 1) = -\frac{1}{2} \cdot (x - 1) \Rightarrow \underline{\underline{y = -\frac{1}{2}x + \frac{1}{2}}}$$

d)  $f: y = \frac{2x-1}{2-x}$ ;  $p: y = 3x$ ;  $p \parallel A$

$$y = \frac{2 \cdot (2-x) - (2x-1) \cdot (-1)}{(2-x)^2} = \frac{4 - 2x + 2x - 1}{(2-x)^2} = \frac{3}{(2-x)^2}$$

$$\frac{3}{(2-x_0)^2} = 3$$

$$3 = 3 \cdot (2-x_0)^2$$

$$(2-x_0)^2 = 1$$

$$(2-x_0)^2 - 1 = 0 \quad [a^2 - b^2 = (a+b) \cdot (a-b)]$$

$$(2-x_0+1) \cdot (2-x_0-1) = 0 \quad x_{01} = 3$$

$$(3-x_0)(1-x_0) = 0 \Rightarrow x_{02} = 1$$

$$\begin{aligned} x_0 = 3 &\Rightarrow y_0 = \frac{2 \cdot 3 - 1}{2 - 3} = \frac{5}{-1} = -5 \\ x_0 = 1 &\Rightarrow y_0 = \frac{2 \cdot 1 - 1}{2 - 1} = \frac{1}{1} = 1 \end{aligned}$$

$$\Rightarrow \begin{array}{l} T_1[3; -5] \\ T_2[1; 1] \end{array}$$

1. dotyčnica:  $y = -5 + \frac{3}{(2-3)^2} \cdot (x-3) = -5 + 3(x-3)$

$$\underline{\underline{y = 3x - 14}}$$

1. normála:  $y = -5 + \frac{-1}{3} \cdot (x-3)$

$$\underline{\underline{y = -\frac{1}{3}x - 4}}$$

2. dotyčnica:  $y = 1 + \frac{3}{(2-1)^2} \cdot (x-1) = 1 + 3 \cdot (x-1)$

$$\underline{\underline{y = 3x - 2}}$$

2. normála:  $y = 1 + \frac{-1}{3} \cdot (x-1)$

$$\underline{\underline{y = -\frac{1}{3}x + \frac{4}{3}}}$$

## Teória: Aplikácie derivácie funkcie:

1. Monotónnosť: pre všetky  $x \in D(f)$  také, že  $f'(x) > 0$  platí, že funkcia rástie  
 pre  $\forall x \in (a, b)$ :  $f'(x) < 0 \Rightarrow f(x)$  na  $(a, b)$  klesá

2. Konvexnosť, konkávnosť: pre  $\forall x \in (a, b)$ :  $f''(x) > 0 \Rightarrow f(x)$  je na  $(a, b)$  konvexná  
 pre  $\forall x \in (a, b)$ :  $f''(x) < 0 \Rightarrow f(x)$  je na  $(a, b)$  konkávná

3. Lokálne extremy: pre  $\forall x_0 \in D(f)$ :  $f'(x_0) = 0 \Rightarrow f(x)$  má v bode  $x_0$  lokálny extrém  
 $x_0$  sa nazýva stacionárny bod  
 ak  $f''(x_0) > 0 \Rightarrow f(x)$  má v  $x_0$  lokálne minimum  
 ak  $f''(x_0) < 0 \Rightarrow f(x)$  má v  $x_0$  lokálne maximum

4. Inflexné body: pre  $\forall x_0 \in D(f)$ :  $f''(x_0) = 0 \Rightarrow f(x)$  ~~smeruje~~ sa mení v  $x_0$  z konvexnej na konkávnu  
 (alebo opačne)  
 $x_0$  sa nazýva inflexný bod

5.1

a)  $f: y = 2x^2 + x - 6$

$f': y' = 4x + 1 > 0 \quad | :1 \rightarrow$  položíme deriváciu  $> 0$  a zistíme, pre ktoré  $x$  to platí

$$4x > -1 \quad | :4$$

$x > -\frac{1}{4} \Rightarrow$  na intervale  $(-\frac{1}{4}; \infty)$  funkcia  $f(x)$  rastie

na opačnom intervale, t.j. na  $(-\infty; -\frac{1}{4})$  funkcia  $f(x)$  klesá

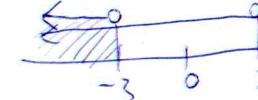
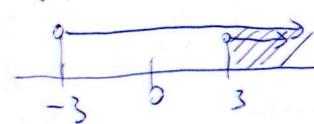
b)  $f: y = x^3 - 27x$

$$y' = 3x^2 - 27 > 0 \quad | :3$$

$$x^2 - 9 > 0$$

$$(x+3)(x-3) > 0 \Rightarrow (x+3 > 0 \wedge x-3 > 0) \vee (x+3 < 0 \wedge x-3 < 0)$$

$$(x > -3 \wedge x > 3) \vee (x < -3 \wedge x < 3)$$



⇒ na  $(-\infty; -3)$  a na  $(3; \infty)$  funkcia rastie  
na  $(-3; 3)$  funkcia klesá

c)  $f: y = 4x^3 + 3x^2 - 6x + 2$

$$y' = 12x^2 + 6x - 6 > 0 \quad | :6$$

$$2x^2 + x - 1 > 0$$

$$D = 1 + 4 \cdot 2 \cdot 1 = 9$$

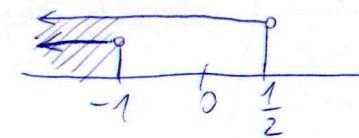
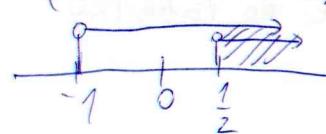
$$x_{1,2} = \frac{-1 \pm \sqrt{9}}{4} = \left\langle \frac{1}{2} \right\rangle$$

$$2x^2 + x - 1 = 2 \cdot (x+1) \cdot (x - \frac{1}{2}) > 0 \quad | :2$$

$$(x+1)(x - \frac{1}{2}) > 0$$

$$(x+1 > 0 \wedge x - \frac{1}{2} > 0) \vee (x+1 < 0 \wedge x - \frac{1}{2} < 0)$$

$$(x > -1 \wedge x > \frac{1}{2}) \vee (x < -1 \wedge x < \frac{1}{2})$$



⇒ na  $(-\infty; -1)$  a na  $(\frac{1}{2}; \infty)$  funkcia rastie  
na  $(-1; \frac{1}{2})$  funkcia klesá

(5.1)

$$d) f: y = 1 - 12x - 9x^2 - 2x^3$$

$$y' = -12 - 18x - 6x^2 > 0 \quad | : (-6)$$

$$x^2 + 3x + 2 < 0$$

$$D = 9 - 4 \cdot 2 = 1$$

$$x_{1,2} = \frac{-3 \pm 1}{2} = \begin{cases} -2 \\ -1 \end{cases}$$

$$\Rightarrow (x+1)(x+2) < 0$$

$$(x < -1 \wedge x > -2) \vee (x > -1 \vee x < -2)$$

na  $(-2; -1)$  funkcia rastie

$\Rightarrow$  na  $(-\infty; -2)$  a na  $(-1; \infty)$  klesá

$$e) f: y = 3 - 2x^2 + 4x^4$$

$$y' = -4x + 16x^3 > 0 \quad | : (+4)$$

$$4x^3 - x > 0$$

$$x \cdot (4x^2 - 1) > 0$$

$$x \cdot (2x-1) \cdot (2x+1) > 0$$

zistíme nulové body pre všetky členy, t.j. keď sa daný člen = 0:

$$x = 0$$

$$2x-1=0$$

$$2x=1$$

$$x = \frac{1}{2}$$

$$2x+1=0$$

$$2x=-1$$

$$x = -\frac{1}{2}$$

|        | $(-\infty; -\frac{1}{2})$ | $(-\frac{1}{2}; 0)$ | $(0; \frac{1}{2})$ | $(\frac{1}{2}; \infty)$ |
|--------|---------------------------|---------------------|--------------------|-------------------------|
| x      | -                         | -                   | +                  | +                       |
| $2x-1$ | -                         | -                   | -                  | +                       |
| $2x+1$ | -                         | +                   | +                  | +                       |
| súčin  | -                         | +                   | -                  | +                       |

v tomto riadku vynášobíme znamienka z daného stĺpca,  
napr.  $\Theta \cdot \Theta \cdot \Theta = \Theta$  atď...  
a zakrúžkujeme tie, kde je  $+$ ,  
lebo tam je súčin  $> 0$

tieto 3 čísla nám rozdelia číselnú os na niekoľko intervalov;  
intervaly zapíšeme do tabuľky

$\rightarrow$  zvolíme si ľubovoľné číslo z daného intervalu a dosadíme ho do daného člena (výrazu); podľa toho aké číslo nám výjde, zapíšeme znakovko tohto výsledku do tabuľky

z 1. intervalu si zvolíme napr. -1000

z 2. si zvolíme -0,25

z 3. zvolíme 0,25

z 4. zvolíme 10

(40)

5.1

e) vidíme, že súčin je  $\oplus$ , teda kladný v dvoch prípadoch  $\Rightarrow$  na  $(-\frac{1}{2}; 0)$  a na  $(\frac{1}{2}; \infty)$  funkcia rastie  
na  $(-\infty; -\frac{1}{2})$  a na  $(0; \frac{1}{2})$  funkcia klesá

$$f: y = \frac{x^5}{5} - \frac{4}{3}x^3 + 1$$

$$y' = \frac{1}{5} \cdot 5x^4 - \frac{4}{3} \cdot 3x^2 = x^4 - 4x^2 > 0$$

$$x^2 \cdot (x^2 - 4) > 0$$

$$x^2 \cdot (x-2) \cdot (x+2) > 0$$

$x^2 \geq 0$  vždy a teda na

znamienko súčinu má vplyv  
 len súčin  ~~$x^2$~~   $(x-2) \cdot (x+2) > 0 \Rightarrow (x > 2 \wedge x > -2) \vee (x < 2 \wedge x < -2)$   
 $(2; \infty) \cup (-\infty; -2)$

$\Rightarrow$  funkcia na  $(-\infty; -2)$  a na  $(2; \infty)$  rastie  
a na  $(-2; 2)$  klesá

$$g) f: y = \frac{x^6}{6} + \frac{x^5}{5}$$

$$y' = \frac{1}{6} \cdot 6x^5 + \frac{1}{5} \cdot 5x^4 = x^5 + x^4 > 0$$

$$x^4 \cdot (x+1) > 0 \rightarrow \text{zase, } x^4 > 0 \text{ vždy} \Rightarrow x+1 > 0$$

$x > -1 \Rightarrow$  na  $(-1; \infty)$  funkcia rastie  
na  $(-\infty; -1)$  funkcia klesá

(41)

5.1

$$h) f: y = \frac{2}{x^2 + 1}$$

$$y' = \frac{-2 \cdot 2x}{(x^2 + 1)^2} > 0 \quad | :(-4)$$

$\frac{x}{(x^2 + 1)^2} < 0 \rightarrow$  menovateľ je vždy  $> 0 \Rightarrow$  zaujíma nás len kedy  $x < 0 \Rightarrow$  na  $(-\infty; 0)$  rastie  
na  $(0; \infty)$  klesá

$$i) f: y = \frac{2x}{x^2 + 1}$$

$$y' = \frac{2 \cdot (x^2 + 1) - 2x \cdot 2x}{(x^2 + 1)^2} = \frac{2x^2 + 2 - 4x^2}{(x^2 + 1)^2} = \frac{-2x^2 + 2}{(x^2 + 1)^2} > 0 \quad | :(-2)$$

$$\frac{(x+1)(x-1)}{(x^2 + 1)^2} < 0 \rightarrow$$

menovateľ je vždy kladný a teda čitatel musí  $< 0$   
 $(x+1)(x-1) < 0$   
 $(x > -1 \wedge x < 1) \vee (x < -1 \wedge x > 1)$   
 $(-1; 1) \quad \emptyset \Rightarrow$  na  $(-1; 1)$  rastie  
na  $(-\infty; -1) \cup (1; \infty)$  klesá

$$j) f: y = \frac{2}{1-x^2}$$

$$y' = \frac{-2 \cdot (-2x)}{(1-x^2)^2} = \frac{4x}{(1-x^2)^2} > 0 \rightarrow$$

menovateľ vždy kladný a teda zistujeme,  
keoby čitatel  $> 0 = 4x > 0 \quad | :4$   
 $x > 0 \Rightarrow$  na  $(0; \infty)$  rastie  
na  $(-\infty; 0)$  klesá

5.1

b)  $f: y = \frac{x-1}{x^2}$

$$y' = \frac{1 \cdot x^2 - (x-1) \cdot 2x}{(x^2)^2} = \frac{x^2 - 2x^2 + 2x}{x^4} = \frac{-x^2 + 2x}{x^4} > 0 \rightarrow \text{menovateľne } \bar{v} \text{ a } y' > 0:$$

$$-x^2 + 2x > 0$$

$$-x \cdot (x-2) > 0 \quad | :(-1)$$

$$x \cdot (x-2) < 0$$

$$(x < 0 \wedge x > 2) \vee (x > 0 \wedge x < 2)$$

$\emptyset$

(0; 2)

$\Rightarrow$  na  $(0; 2)$  rastie

na  $(-\infty; 0) \cup (2; \infty)$  klesá

c)  $f: y = \frac{2}{x} + \frac{x}{2} = 2 \cdot \frac{1}{x} + \frac{1}{2} \cdot x = 2x^{-1} + \frac{1}{2}x$

$$y' = -2x^{-2} + \frac{1}{2} = \frac{-2}{x^2} + \frac{1}{2} \quad (\cancel{\text{zložka}}) = \frac{-4+x^2}{2x^2} = \frac{x^2-4}{2x^2} > 0 \rightarrow \text{menovateľne } > 0 \quad \forall x$$

~~(zložka menovateľne)~~

$$(x+2)(x-2) > 0$$

$$(x > -2 \wedge x > 2) \vee (x < -2 \wedge x < 2)$$

(2;  $\infty$ )

$\cup$

( $-\infty; -2$ )  $\Rightarrow$  na  $(-\infty; -2) \cup (2; \infty)$  rastie

na  $(-2; 2)$  klesá

m)  $f: y = \frac{x^2}{2-x}$

$$y' = \frac{2x \cdot (2-x) - x^2 \cdot (-1)}{(2-x)^2} = \frac{4x - 2x^2 + x^2}{(2-x)^2} = \frac{-x^2 + 4x}{(2-x)^2} > 0 \rightarrow \text{menovateľne } > 0 \quad \forall x$$

$$-x^2 + 4x > 0$$

$$-x \cdot (x-4) > 0 \quad | :(-1)$$

$$x \cdot (x-4) < 0$$

$$(x < 0 \wedge x > 4) \vee (x > 0 \wedge x < 4)$$

$\emptyset$

(0; 4)

$\Rightarrow$  na  $(0; 4)$  rastie

na  $(-\infty; 0) \cup (4; \infty)$  klesá

5.1

$$n) f: y = x + \frac{x}{x^2 - 1} = \frac{x \cdot (x^2 - 1) + x}{x^2 - 1} = \frac{x \cdot [(x^2 - 1) + 1]}{x^2 - 1} = \frac{x^3}{x^2 - 1}$$

$$y' = \frac{3x^2(x^2 - 1) - x^3 \cdot 2x}{(x^2 - 1)^2} = \frac{3x^4 - 3x^2 - 2x^4}{(x^2 - 1)^2} = \frac{x^4 - 3x^2}{(x^2 - 1)^2} > 0 \Rightarrow$$

menovateľ  $> 0 \quad \forall x$ 

$$x^4 - 3x^2 > 0$$

$$x^2 \cdot (x^2 - 3) > 0 \Rightarrow x^2 > 0 \quad \forall x$$

$$(x + \sqrt{3})(x - \sqrt{3}) > 0$$

$$(x > -\sqrt{3} \wedge x > \sqrt{3}) \vee (x < -\sqrt{3} \wedge x < \sqrt{3})$$

$$(\sqrt{3}; \infty) \quad \cup \quad (-\infty; -\sqrt{3})$$

$\Rightarrow$  na  $(-\infty; -\sqrt{3})$  a na  $(\sqrt{3}; \infty)$  funkcia rastie  
na  $(-\sqrt{3}; \sqrt{3})$  funkcia klesá

5.2

$$a) f: y = \sqrt{1-x} = (1-x)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(1-x)^{-\frac{1}{2}} \cdot (-1) = \frac{-1}{2\sqrt{1-x}} > 0$$

odmocnina je vždy  $> 0 \Rightarrow$  menovateľ  $> 0$   
 zároveň čitatel  $= -1 < 0$  vždy

zlomok je vždy záporný a funkcia klesá na celom  $D(f)$

$$b) f: y = \sqrt[3]{(x+2)^2} = (x+2)^{\frac{2}{3}}$$

$$y' = \frac{2}{3}(x+2)^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x+2}} > 0 \Rightarrow 2 > 0 \text{ vždy a teda zistujeme, kedy menovateľ } > 0:$$

$$3 \cdot \sqrt[3]{x+2} > 0 / :3$$

$$\sqrt[3]{x+2} > 0 / 3$$

$$x+2 > 0$$

$x > -2$   $\Rightarrow$  na  $(-2; \infty)$  rastie

na  $(-\infty; -2)$  klesá

$$c) f: y = \sqrt{\frac{1+x}{1-x}} = \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}} ; D(f): \frac{1+x}{1-x} \geq 0 \wedge \frac{1-x+0}{x+1}$$

$$(1+x \geq 0 \wedge 1-x > 0) \vee (1+x \leq 0 \wedge 1-x < 0)$$

$$(x \geq -1 \wedge x < 1) \vee (x \leq -1 \wedge x > 1)$$

$$<-1; 1)$$

$$\Rightarrow D(f) = [-1; 1]$$

$$y' = \frac{1}{2} \cdot \left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} \cdot \frac{1 \cdot (1-x) - (1+x) \cdot (-1)}{(1-x)^2} = \frac{1}{2} \cdot \left(\frac{1+x}{1-x}\right)^{-\frac{1}{2}} \cdot \frac{1-x+1+x}{(1-x)^2} = \frac{1}{2} \cdot \sqrt{\frac{1-x}{1+x}} \cdot \frac{2}{(1-x)^2} > 0 \text{ vždy,}$$

lebo  $\sqrt{\square} > 0 \wedge (1-x)^2 > 0$

$$\boxed{\text{vzorec: } \left(\frac{a}{b}\right)^{-1} = \frac{b}{a}}$$

$\Rightarrow$  funkcia rastie na  $<-1; 1)$

(5.2) d)  $f: y = (x-3)\sqrt{x}$ ;  $D(f) = \langle 0; \infty \rangle$  lebo  $x \geq 0$

$$y' = 1 \cdot x^{\frac{1}{2}} + (x-3) \cdot \frac{1}{2}x^{-\frac{1}{2}} = \sqrt{x} + \frac{x-3}{2\sqrt{x}} = \frac{2x+x-3}{2\sqrt{x}} = \frac{3x-3}{2\sqrt{x}} > 0 \rightarrow$$

menovateľ výrazy  $> 0$   
 $3x-3 > 0 \quad | :3$   
 $x-1 > 0 \quad | +1$

$\boxed{x > 1}$

$\Rightarrow$  funkcia rastie na  $(1; \infty)$   
klesá na  $\langle 0; 1 \rangle$

e)  $f: y = x\sqrt{1-x}$ ;  $D(f) = 1-x \geq 0$   
 $x \leq 1 \Rightarrow D(f) = (-\infty; 1]$

$$y' = [x \cdot (1-x)^{\frac{1}{2}}]' = 1 \cdot (1-x)^{\frac{1}{2}} + x \cdot \frac{1}{2} \cdot (1-x)^{-\frac{1}{2}} \cdot (-1) = \sqrt{1-x} - \frac{1}{2}x \cdot \frac{1}{\sqrt{1-x}} = \sqrt{1-x} - \frac{x}{2\sqrt{1-x}} = \frac{2(1-x)-x}{2\sqrt{1-x}} =$$
 $= \frac{2-2x-x}{2\sqrt{1-x}} = \frac{2-3x}{2\sqrt{1-x}} > 0 \Leftrightarrow 2-3x > 0$ 

$3x < 2$   
 $\boxed{x < \frac{2}{3}} \Rightarrow$  na  $(-\infty; \frac{2}{3})$  rastie  
na  $(\frac{2}{3}; 1)$  klesá

f)  $f: y = x\sqrt{1+x^2}$ ;  $D(f) = 1+x^2 \geq 0$  platí výrazy  $\Rightarrow D(f) = (-\infty; \infty)$

$$y' = [x \cdot (1+x^2)^{\frac{1}{2}}]' = 1 \cdot (1+x^2)^{\frac{1}{2}} + x \cdot \frac{1}{2} \cdot (1+x^2)^{-\frac{1}{2}} \cdot 2x = \sqrt{1+x^2} + \frac{x^2}{\sqrt{1+x^2}} \rightarrow$$

všetky členy sú výrazy  $\geq 0$   
 $\Rightarrow$  funkcia rastie na  $(-\infty; \infty)$

(46)

(5.2)

$$g) y = \frac{x-3}{\sqrt{1+x^2}} = \frac{x-3}{(1+x^2)^{\frac{1}{2}}} ; D(f) = \mathbb{R} = (-\infty; \infty)$$

$$y' = \frac{1 \cdot (1+x^2)^{\frac{1}{2}} - (x-3) \cdot \frac{1}{2} \cdot (1+x^2)^{-\frac{1}{2}} \cdot 2x}{[(1+x^2)^{\frac{1}{2}}]^2} = \frac{\sqrt{1+x^2} - x(x-3) \cdot \frac{1}{\sqrt{1+x^2}}}{(1+x^2)} = \frac{\sqrt{1+x^2} - \frac{x(x-3)}{\sqrt{1+x^2}}}{1+x^2} > 0 \Rightarrow \text{menovatel} > 0 \Rightarrow \text{václivý}$$

$$\Rightarrow \sqrt{1+x^2} - \frac{x(x-3)}{\sqrt{1+x^2}} > 0$$

$$\frac{1+x^2 - x(x-3)}{\sqrt{1+x^2}} = \frac{1+x^2 - x^2 + 3x}{\sqrt{1+x^2}} = \frac{3x+1}{\sqrt{1+x^2}} > 0 \Leftrightarrow 3x+1 > 0$$

$x > -\frac{1}{3} \Rightarrow \text{rastie na } (-\frac{1}{3}; \infty)$   
Klesá na  $(-\infty; -\frac{1}{3})$

$$h) f: y = x \cdot \sqrt{4-x^2} = x \cdot (4-x^2)^{\frac{1}{2}} ; D(f): 4-x^2 \geq 0$$

$$(2-x)(2+x) \geq 0$$

$$(2-x \geq 0 \wedge 2+x \geq 0) \vee (2-x \leq 0 \wedge 2+x \leq 0) \Rightarrow D(f) = \langle -2; 2 \rangle$$

(~~zjednodušenie funkcie~~)

$$y' = 1 \cdot (4-x^2)^{\frac{1}{2}} + x \cdot \frac{1}{2} (4-x^2)^{-\frac{1}{2}} \cdot (-2x) = \sqrt{4-x^2} - \frac{x^2}{\sqrt{4-x^2}} = \frac{4-x^2-x^2}{\sqrt{4-x^2}} = \frac{4-2x^2}{\sqrt{4-x^2}} > 0 \Leftrightarrow$$

$$\Leftrightarrow 4-2x^2 > 0$$

$$2x^2 < 4$$

$$x^2 < 2 \Rightarrow x \in (-\sqrt{2}; \sqrt{2}) \Rightarrow$$

$$(\sqrt{2}-x)(\sqrt{2}+x) > 0$$

rastie na  $(-\sqrt{2}; \sqrt{2})$

Klesá na  $\langle -2; -\sqrt{2} \rangle \text{ a na } (\sqrt{2}; 2)$

(5.2)

$$i) f: y = \frac{2}{3}x + \sqrt[3]{x^2} = \frac{2}{3}x + x^{\frac{2}{3}}; D(f) = \mathbb{R}$$

$$y' = \frac{2}{3} + \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3} \cdot \left(1 + \frac{1}{\sqrt[3]{x}}\right) > 0 \quad | : \frac{2}{3}$$

$$1 + \frac{1}{\sqrt[3]{x}} > 0$$

$$\frac{\sqrt[3]{x} + 1}{\sqrt[3]{x}} > 0 \Leftrightarrow (\sqrt[3]{x} + 1 > 0 \wedge \sqrt[3]{x} > 0) \vee (\sqrt[3]{x} + 1 < 0 \wedge \sqrt[3]{x} < 0)$$

$$(\sqrt[3]{x} > -1 \wedge \sqrt[3]{x} > 0) \vee (\sqrt[3]{x} < -1 \wedge \sqrt[3]{x} < 0)$$

$$\sqrt[3]{x} > 0$$

$$x > 0$$

$$\sqrt[3]{x} < -1$$

$x < -1 \Rightarrow$  na  $(-\infty; -1) \cup (0; \infty)$  rastie

na  $(-1; 0)$  klesá

$$ii) f: y = 2x - 3 \cdot \sqrt[3]{(1-x)^2} = 2x - 3 \cdot (1-x)^{\frac{2}{3}}; D(f) = \mathbb{R}$$

$$y' = 2 - 3 \cdot \frac{2}{3} \cdot (1-x)^{-\frac{1}{3}} \cdot (-1) = 2 + \frac{2}{\sqrt[3]{1-x}} \quad (\text{_____}) > 0$$

$$\frac{2}{\sqrt[3]{1-x}} > -2 \quad | : 2$$

$$\frac{1}{\sqrt[3]{1-x}} > -1$$

$$\frac{1}{\sqrt[3]{1-x}} + 1 > 0$$

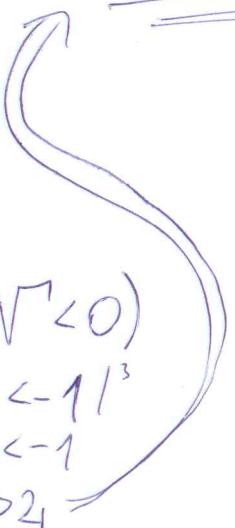
$$\frac{1 + \sqrt[3]{1-x}}{\sqrt[3]{1-x}} > 0 \Leftrightarrow (1 + \sqrt[3]{1-x} > 0 \wedge \sqrt[3]{1-x} > 0) \vee (1 + \sqrt[3]{1-x} < 0 \wedge \sqrt[3]{1-x} < 0)$$

$$\sqrt[3]{1-x} > 0 \quad |^3$$

$$1-x > 0 \\ \boxed{x < 1}$$

na  $(-\infty; 1) \cup (2; \infty)$  rastie

na  $(1; 2)$  klesá



(47)

párové odmociny existujú len z nezáporných čísel  
 $(\sqrt[4]{4}; \sqrt[4]{5}; \dots)$

nepárové odmociny existujú zo všetkých čísel  $(\sqrt[3]{7}; \sqrt[5]{-2}; \dots)$

5.2

k)  $f: y = x + e^{-x}$ ;  $D(f) = \mathbb{R}$

$$y' = 1 + e^{-x} \cdot (-1) = 1 - e^{-x} > 0$$

$$\ln e^{-x} < \ln 1 \quad | \text{zlogaritmujeme}$$

$$\ln e^{-x} < \ln 1 \rightarrow \boxed{\log_a e^{-x} = x; \log_a 1 = 0 \quad \forall a \in \mathbb{R}} !$$

$$-x < 0$$

$$\underline{x > 0} \Rightarrow \text{na } (0; \infty) \text{ rastie}$$

$$\text{na } (-\infty; 0) \text{ klesá}$$

l)  $f: y = 2^{x^2-6x+2}$ ;  $D(f) = \mathbb{R}$

$$y' = 2^{x^2-6x+2} \cdot \ln 2 \cdot (2x-6) \rightarrow 2 \text{ umocnené na hocico} > 0$$

$$\ln 2 \approx 0,7 > 0$$

$$\downarrow > 0 \Leftrightarrow 2x-6 > 0 \quad \text{na } (3; \infty) \text{ rastie}$$

$$2x > 6$$

$$\underline{x > 3} \Rightarrow \text{na } (-\infty; 3) \text{ klesá}$$

m)  $f: y = \frac{e^{2x-x^2}}{2} = \frac{1}{2} e^{2x-x^2}$ ;  $D(f) = \mathbb{R}$

$$y' = \frac{1}{2} \cdot e^{2x-x^2} \cdot (2-2x) > 0 \Leftrightarrow 2-2x > 0$$

$$\underline{x < 1} \Rightarrow \text{na } (-\infty; 1) \text{ rastie}$$

$$\text{na } (1; \infty) \text{ klesá}$$

(5.2) a)  $f: y = x \cdot e^x$ ;  $D(f) = \mathbb{R}$

$$y' = 1 \cdot e^x + x \cdot e^x = e^x \cdot (x+1) > 0 \Leftrightarrow x+1 > 0 \Rightarrow$$

$x > -1$

na  $(-1; \infty)$  rastie  
na  $(-\infty; -1)$  klesá

(5.3) a)  $f: y = x \cdot e^{\frac{1}{x}}$ ;  $D(f) = (-\infty; 0) \cup (0; \infty)$

$$y' = 1 \cdot e^{\frac{1}{x}} + x \cdot e^{\frac{1}{x}} \cdot (-x^{-2}) = e^{\frac{1}{x}} + e^{\frac{1}{x}} \cdot \frac{x}{-x^2} = e^{\frac{1}{x}} \cdot \left(1 - \frac{1}{x}\right) > 0 \Leftrightarrow$$

$\frac{x-1}{x} > 0$

$(x-1 > 0 \wedge x > 0) \vee (x-1 < 0 \wedge x < 0)$   
 $(1; \infty) \quad (-\infty; 0)$

$\Rightarrow$  na  $(-\infty; 0)$  a na  $(1; \infty)$  rastie  
na  $(0; 1)$  klesá

b)  $f: y = x \cdot e^{-\frac{x^2}{2}}$ ;  $D(f) = \mathbb{R}$

$$y' = 1 \cdot e^{-\frac{x^2}{2}} + x \cdot e^{-\frac{x^2}{2}} \cdot \left(-\frac{1}{2} \cdot 2x\right) = e^{-\frac{x^2}{2}} \cdot (1-x^2) > 0 \Leftrightarrow (1+x)(1-x) > 0$$

$\frac{1}{e^{-\frac{x^2}{2}}} > 0$

$(1+x > 0 \wedge 1-x > 0) \vee (1+x < 0 \wedge 1-x < 0)$   
 $(x > 1 \wedge x < 1) \vee (x < -1 \wedge x > 1)$   
 $(-1; 1) \quad \emptyset$

$\Rightarrow$  na  $(-1; 1)$  rastie  
na  $(-\infty; -1)$  a na  $(1; \infty)$  klesá

5.3

c)  $f: y = (1+x^2) \cdot e^{-x^2}$  ;  $D(f) = \mathbb{R}$

$$y' = 2x \cdot (e^{-x^2}) + (1+x^2) \cdot e^{-x^2} \cdot (-2x) = e^{-x^2} \cdot (2x + (1+x^2) \cdot (-2x)) = 2x \cdot e^{-x^2} \cdot (1 - 1 - x^2) = 2x \cdot e^{-x^2} \cdot (-x) > 0 \Leftrightarrow \begin{cases} x > 0 \\ x < 0 \end{cases}$$

$\Rightarrow$  na  $(-\infty; 0)$  rastie  
na  $(0; \infty)$  klesá

d)  $f: y = \frac{e^x}{x+1}$  ;  $D(f) = \mathbb{R} - \{-1\} = (-\infty; -1) \cup (-1; \infty)$

$$y' = \frac{e^x \cdot (x+1) - e^x \cdot 1}{(x+1)^2} = \frac{e^x \cdot (x+1-1)}{(x+1)^2} = \frac{e^x}{(x+1)^2} \cdot x > 0 \Leftrightarrow x > 0 \Rightarrow \begin{cases} \text{na } (0; \infty) \text{ rastie} \\ \text{na } (-\infty; -1) \text{ a na } (-1; 0) \text{ klesá} \end{cases}$$

e)  $f: y = x - 2 \cdot \ln x$  ;  $D(f) = (0; \infty)$

$$y' = 1 - 2 \cdot \frac{1}{x} = \frac{x-2}{x} > 0 \Leftrightarrow x-2 > 0 \quad \begin{cases} x > 2 \\ x > 0 \end{cases} \Rightarrow \begin{cases} \text{na } (2; \infty) \text{ rastie} \\ \text{na } (0; 2) \text{ klesá} \end{cases}$$

f)  $f: y = 2x^2 - \ln x$  ;  $D(f) = (0; \infty)$

$$y' = 4x - \frac{1}{x} = \frac{4x^2 - 1}{x} \Leftrightarrow 4x^2 - 1 > 0 \Leftrightarrow \underbrace{(2x-1)(2x+1)}_{> 0} > 0 \Leftrightarrow 2x-1 > 0 \Leftrightarrow x > \frac{1}{2} \Rightarrow \begin{cases} \text{na } (\frac{1}{2}; \infty) \text{ rastie} \\ \text{na } (0; \frac{1}{2}) \text{ klesá} \end{cases}$$

$\forall x \in D(f) \Rightarrow x > 0 \Rightarrow 2x > 0 \Rightarrow 2x+1 > 0$

5.3

g)  $f: y = \ln(1-x)$ ;  $D(f): 1-x > 0$   
 $x < 1 \Rightarrow D(f) = (-\infty; 1)$

$$y' = \frac{1}{1-x} \cdot (-1) = \frac{-1}{1-x} \rightsquigarrow \begin{cases} -1 < 0 \text{ vtedy} \\ 1-x > 0 \text{ z } D(f) \end{cases} \quad \left. \begin{array}{l} \frac{-1}{1-x} < 0 \text{ vtedy} \\ \Rightarrow \text{klesá na } (-\infty; 1), \text{ t.j. na celom } D(f) \end{array} \right\}$$

h)  $f: y = \ln(x^2 - 4)$ ;  $D(f): x^2 - 4 > 0$   
 $x^2 > 4 \Rightarrow D(f) = (-\infty; -2) \cup (2; \infty)$   
 $(x-2)(x+2) > 0$   
 $\dots$

$$y' = \frac{1}{x^2-4} \cdot 2x = \frac{2x}{x^2-4} > 0 \Leftrightarrow \begin{cases} 2x > 0, \text{ lebo } x^2-4 \text{ je vtedy} > 0 \text{ (D(f))} \\ x > 0 \end{cases} \Rightarrow \begin{array}{l} \text{na } (2; \infty) \text{ rastie} \\ \text{na } (-\infty; -2) \text{ klesá} \end{array}$$

i)  $f: y = x \cdot \ln x$ ;  $D(f) = (0; \infty)$

$$y' = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1 > 0$$

$\ln x > -1 \rightsquigarrow$  obe strany nerovnice díme do exponentu e

$$e^{\ln x} > e^{-1} \Leftrightarrow \boxed{e^{\log_e x} = x}$$

$$\boxed{x > \frac{1}{e}} \Rightarrow \begin{array}{l} \text{na } (\frac{1}{e}; \infty) \text{ rastie} \\ \text{na } (0; \frac{1}{e}) \text{ klesá} \end{array}$$

5.3

j)  $f: y = \frac{\ln x}{x}$  ;  $D(f) = (0; \infty)$

$$y' = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2} > 0 \Leftrightarrow 1 - \ln x > 0$$

$\ln x < 1$   
 $e^{\ln x} < e^1$   
 $x < e$

na  $(0; e)$  rastie  
na  $(e; \infty)$  klesá

k)  $f: y = \frac{\ln(3-x)}{3-x}$  ;  $D(f) = 3-x > 0$   
 $x < 3 \Rightarrow D(f) = (-\infty; 3)$

$$y' = \frac{\frac{1}{3-x} \cdot (-1) \cdot (3-x) - \ln(3-x) \cdot (-1)}{(3-x)^2} = \frac{-1 + \ln(3-x)}{(3-x)^2} > 0 \Leftrightarrow -1 + \ln(3-x) > 0$$

$\ln(3-x) > 1$   
 $3-x > e$   
 $x < 3-e$

na  $(-\infty; 3-e)$  rastie  
na  $(3-e; 3)$  klesá

l)  $f: y = \ln\left(\frac{x+2}{2-x}\right)$  ;  $D(f): \frac{x+2}{2-x} > 0 \Leftrightarrow (x+2 > 0 \wedge 2-x > 0) \vee (x+2 < 0 \wedge 2-x < 0)$   
 $(x > -2 \wedge x < 2) \vee (x < -2 \wedge x > 2) \Rightarrow D(f) = (-2; 2)$

$$y' = \frac{1}{\frac{x+2}{2-x}} \cdot \frac{1 \cdot (2-x) - (x+2) \cdot (-1)}{(2-x)^2} = \frac{2-x}{x+2} \cdot \frac{2-x+x+2}{(2-x)^2} = \frac{2-x}{x+2} \cdot \frac{4}{\underbrace{(2-x)^2}_{>0}} > 0 \Leftrightarrow \frac{2-x}{x+2} > 0 \Leftrightarrow$$
 $\Leftrightarrow (2-x > 0 \wedge x+2 > 0) \vee (2-x < 0 \wedge x+2 < 0)$ 
 $(x < 2 \wedge x > -2) \vee (x > 2 \wedge x < -2) \Rightarrow$ 

na  $(-2; 2)$  rastie

(53) m)  $f: y = \frac{2}{x} + \ln x^2$  ;  $D(f) = (-\infty; 0) \cup (0; \infty)$

$$y' = -2x^{-2} + \frac{1}{x^2} \cdot 2x = \frac{-2}{x^2} + \frac{2}{x} = \frac{-2+2x}{x^2} = \frac{2x-2}{x^2} > 0 \Leftrightarrow 2x-2 > 0 /:2$$

$$\begin{array}{l} x-1 > 0 \\ x > 1 \end{array} \Rightarrow \text{na } (1; \infty) \text{ rastie}$$

na  $(-\infty; 0)$  a na  $(0; 1)$  klesá

m)  $f: y = \frac{x}{\ln x}$  ;  $D(f) = \underbrace{x > 0}_{x \neq 1} \wedge \ln x \neq 0 \Rightarrow D(f) = (0; 1) \cup (1; \infty)$

$$y' = \frac{1 \cdot \ln x - x \cdot \frac{1}{x}}{\ln^2 x} = \frac{\ln x - 1}{\ln^2 x > 0} > 0 \Leftrightarrow \ln x - 1 > 0$$

$$\begin{array}{l} \ln x > 1 \\ x > e \end{array} \Rightarrow \begin{array}{l} \text{na } (e; \infty) \text{ rastie} \\ \text{na } (0; 1) \text{ a na } (1; e) \text{ klesá} \end{array}$$

(6.1) a)  $f: y = x^3 - 9x^2 + 1$

$$y' = 3x^2 - 18x$$

$$y'' = 6x - 18 = 0$$

$$6x = 18$$

$x = 3 \rightarrow$  inflexný bod

$$y'' > 0$$

$$6x - 18 > 0$$

$x > 3 \Rightarrow$  funkcia konvexná, inak konkávná

$\Rightarrow$  na  $(3; \infty)$  konvexná

na  $(-\infty; 3)$  konkávná

$x_0 = 3$  je inflexný bod

6.1

b)  $f: y = x^4 - 2x^3 - 7$

$y' = 4x^3 - 6x^2$

$y'' = 12x^2 - 12x = 0 \quad | :12$

$x(x-1) = 0 \rightarrow \boxed{x_0 = 0 \wedge x_0 = 1}$

$y'' > 0$

$12x^2 - 12x > 0$

$x(x-1) > 0 \Leftrightarrow (x > 0 \wedge x > 1) \vee (x < 0 \wedge x < 1)$ 
 $(1; \infty) \cup (-\infty; 0)$

$\Rightarrow x_0 = 0 \text{ a } x_0 = 1$  sú inflexné body funkcie

funkcia je na  $(-\infty; 0)$  a na  $(1; \infty)$  konkávná  
funkcia je na  $(0; 1)$  konvexná

c)  $f: y = x^4 + 4x^3 - 18x^2 + 3x + 2$

$y' = 4x^3 + 12x^2 - 36x + 3$

$y'' = 12x^2 + 24x - 36 = 0 \quad | :12$ 
 $x^2 + 2x - 3 = 0$

$D = 4 + 4 \cdot 3 = 16$

$x_{1,2} = \frac{-2 \pm 4}{2} = \begin{cases} -3 \\ 1 \end{cases} \Rightarrow \boxed{x_0 = -3 \wedge x_0 = 1}$

$y'' > 0$

$x^2 + 2x - 3 > 0$

$(x+3)(x-1) > 0$

$(x > -3 \wedge x > 1) \vee (x < -3 \wedge x < 1)$ 
 $(1; \infty) \cup (-\infty; -3)$

$\Rightarrow$  inflexné body:  $x_0 = -3 \wedge x_0 = 1$

konvexná na  $(-\infty; -3) \wedge$  na  $(1; \infty)$

konkávná na  $(-3; 1)$

(6.1) d)  $f: y = x^4 - x^5$

$$y' = 4x^3 - 5x^4$$

$$y'' = 12x^2 - 20x^3 = 0 \quad | :4$$

$$x^2(3 - 5x) = 0 \Leftrightarrow x^2 = 0 \vee 3 - 5x = 0$$

$$\boxed{x_0 = 0 \vee x_0 = \frac{3}{5}}$$

$$x^2(3 - 5x) > 0 \Leftrightarrow 3 - 5x > 0 \Leftrightarrow x < \frac{3}{5} \quad [x^2 > 0 \text{ vtedy}]$$

$$\Rightarrow \text{inflexné body: } x_0 = 0 \wedge x_0 = \frac{3}{5}$$

konvexná na  $(-\infty; 0)$  a na  $(0; \frac{3}{5})$

konkávná na  $(\frac{3}{5}; \infty)$



e)  $f: y = \frac{x^6}{6} - \frac{x^5}{4} + 3$

$$y' = \frac{1}{6} \cdot 6x^5 - \frac{1}{4} \cdot 5x^4 = x^5 - \frac{5}{4}x^4$$

$$y'' = 5x^4 - \frac{5}{4} \cdot 4x^3 = 5x^4 - 5x^3 = 5x^3(x-1) = 0 \Leftrightarrow \boxed{x_0 = 0 \vee x_0 = 1}$$

$$5x^3(x-1) > 0 \Leftrightarrow \underbrace{5x^2}_{>0} \cdot x \cdot (x-1) > 0 \Leftrightarrow x(x-1) > 0$$

$$(x > 0 \wedge x > 1) \vee (x < 0 \wedge x < 1) \\ (1; \infty) \cup (-\infty; 0)$$

$$\Rightarrow \text{ib: } x_0 = 0 \wedge x_0 = 1$$

konvexná na  $(-\infty; 0)$  a na  $(1; \infty)$

konkávná na  $(0; 1)$

f)  $f: y = 3x - (4-x)^5$

$$y' = (y) = [3 - 5 \cdot (4-x)^4 \cdot (-1)] = (3 + 5 \cdot (4-x)^4) = 20 \cdot (4-x)^3 \cdot (-1) = -20 \cdot (4-x)^3 = 0 \Leftrightarrow \boxed{x_0 = 4}$$

$$y'' > 0 \Leftrightarrow -20(4-x)^3 > 0 \Leftrightarrow \underbrace{-20 \cdot (4-x)^2}_{>0} \cdot (4-x) > 0 \Leftrightarrow -(4-x) > 0 \Leftrightarrow x-4 > 0 \Leftrightarrow \boxed{x > 4}$$

$$\Rightarrow \text{ib: } x_0 = 4$$

konvexná na  $(4; \infty)$

konkávná na  $(-\infty; 4)$

(6.1) g)  $f: y = x^4 + 2x^3 + 6x^2$

 $y'' = (4x^3 + 6x^2 + 12x)' = 12x^2 + 12x + 12 = 0$ 
 $x^2 + x + 1 = 0$ 
 $D = 1 - 4 = -3 \Rightarrow \exists \text{ inflexný bod}$ 

postupujeme tak, že dosadíme do vztahu  
 ľubovoľné číslo a zistíme, aké znamienko  
 má výsledok; platí: ak  $D < 0 \Rightarrow$  funkcia je  
 $\bar{v}ad>0$  alebo  $<0$ , teda aké znamienko  
 bude mať výsledok, také znamienko má funkcia  
 po dosadení všetkých  $x \in D(f)$ :

 $x^2 + x + 1 \rightarrow \text{dosadíme}, D = 0^2 + 0 + 1 = 1 > 0$ 
 $\Rightarrow y'' > 0 \quad \forall x \in D(f) \Rightarrow \text{ib } \exists$ 

funkcia je konkávná na  $(-\infty; \infty)$

h)  $f: y = x + \frac{1}{x} ; D(f) = (-\infty; 0) \cup (0; \infty)$

 $y''' = (1 - x^{-2})' = 2x^{-3} = \frac{2}{x^3} = 0 \rightarrow \text{neexistuje nukdy}$ 
 $\Rightarrow \exists \text{ ib}$

$\frac{2}{x^3} > 0 \Leftrightarrow x^3 > 0 \Leftrightarrow x > 0,$ 
 $\Rightarrow \text{ib } \exists$ 

konkávná na  $(0; \infty)$   
konvexná na  $(-\infty; 0)$

i)  $f: y = 3x + \frac{1}{2x^2} ; D(f) = (-\infty; 0) \cup (0; \infty)$

 $y''' = \left(3x + \frac{1}{2} \cdot x^{-2}\right)' = \left(3 + \frac{1}{2} \cdot (-2)x^{-3}\right)' = 3x^{-4} = \frac{3}{x^4} \neq 0 \text{ nukdy} ; \frac{3}{x^4} > 0 \quad \bar{v}ad \Rightarrow \text{ib } \exists$ 

konvexná na celom  $D(f)$

j)  $f: y = \frac{3x^2}{1-x} ; D(f) = (-\infty; 1) \cup (1; \infty)$

 $y' = \frac{6x \cdot (1-x) - 3x^2 \cdot (-1)}{(1-x)^2} = \frac{6x - 6x^2 + 3x^2}{(1-x)^2} = \frac{6x - 3x^2}{(1-x)^2}$ 
 $y''' = \frac{(6-6x) \cdot (1-x)^2 - (6x-3x^2) \cdot 2 \cdot (1-x) \cdot (-1)}{(1-x)^4} = \frac{6 \cdot (1-x)^3 - 3x(2-x) \cdot (-2) \cdot (1-x)}{(1-x)^4} = \frac{(1-x) \cdot [6(1-x)^2 + 6(2-x)]}{(1-x)^4} = \textcircled{*}$

$$\textcircled{2} = \frac{6 \cdot (1-2x+x^2+2-x)}{(1-x)^3} = \frac{6 \cdot (x^2-3x+3)}{(1-x)^3} \neq 0 \quad \text{nikdy, lebo } x^2-3x+3 \neq 0, \text{ lebo } D=9-4 \cdot 3 < 0$$

dosaďme  $\varrho_i = 0^2 - 3 \cdot 0 + 3 > 0 \quad \forall x \in D(f)$

$$\frac{6(x^2-3x+3)}{(1-x)^3} > 0 \Leftrightarrow \underbrace{\frac{6(x^2-3x+3)}{(1-x)^2}}_{{>}0} \cdot \frac{1}{1-x} > 0 \Leftrightarrow \frac{1}{1-x} > 0 \Leftrightarrow 1-x > 0 \Leftrightarrow x < 1$$

$\Rightarrow$  ib  $\exists$

konvexná na  $(-\infty; 1)$

konkávná na  $(1; \infty)$

$$\textcircled{6.1} \text{ b) } f: y = \frac{x^2+x+21}{x+2} ; \boxed{D(f) = (-\infty; -2) \cup (-2; \infty)}$$

$$y' = \frac{(2x+1)(x+2) - (x^2+x+21) \cdot 1}{(x+2)^2} = \frac{2x^2+4x+x+2 - x^2 - x - 21}{(x+2)^2} = \frac{x^2+4x-19}{(x+2)^2}$$

$$y'' = \frac{(2x+4) \cdot 2(x+2)^2 - (x^2+4x-19) \cdot 2(x+2)}{(x+2)^4} = \frac{2 \cdot (x+2)^3 - 2(x^2+4x-19)(x+2)}{(x+2)^4} = \frac{2(x+2) \cdot [(x+2)^2 - (x^2+4x-19)]}{(x+2)^4} =$$

$$= \frac{2 \cdot (x^2+4x+4 - x^2 - 4x + 19)}{(x+2)^3} = \frac{46}{(x+2)^3} \neq 0 \quad \text{nikdy} ; \quad \frac{46}{(x+2)^3} > 0 \Leftrightarrow \underbrace{\frac{46}{(x+2)^2}}_{>0} \cdot \frac{1}{x+2} > 0 \Leftrightarrow x+2 > 0 \Leftrightarrow x > -2$$

$\Rightarrow$  ib  $\exists$

konvexná na  $(-2; \infty)$

konkávná na  $(-\infty; -2)$

(6.1) l)  $f: y = \frac{2x}{1+x^2}$  ;  $D(f) = \mathbb{R}$

$$y' = \frac{2 \cdot (1+x^2) - 2x \cdot 2x}{(1+x^2)^2} = \frac{2+2x^2-4x^2}{(1+x^2)^2} = \frac{2-2x^2}{(1+x^2)^2}$$

$$y'' = \frac{(-4x)(1+x^2)^2 - (2-2x^2) \cdot 2 \cdot (1+x^2) \cdot 2x}{(1+x^2)^4} = \frac{(1+x^2) \cdot [-4x(1+x^2) - 4x(2-2x^2)]}{(1+x^2)^4} = \frac{-4x-4x^3-8x+8x^3}{(1+x^2)^3} = \frac{4x^3-12x}{(1+x^2)^3} =$$

$$= \frac{4x(x^2-3)}{(1+x^2)^3} = 0 \Leftrightarrow \underbrace{x_0=0}_{x_0=\pm\sqrt{3}} \wedge x^2-3=0$$

$$\frac{4x(x^2-3)}{(1+x^2)^3} > 0 \Leftrightarrow \underbrace{\frac{4}{(1+x^2)^3}}_{>0} \cdot x(x^2-3) > 0 \Leftrightarrow$$

$$\Leftrightarrow x(x-\sqrt{3})(x+\sqrt{3}) > 0$$

|              | $(-\infty; -\sqrt{3})$ | $(-\sqrt{3}; 0)$ | $(0; \sqrt{3})$ | $(\sqrt{3}; \infty)$ |
|--------------|------------------------|------------------|-----------------|----------------------|
| $x$          | -                      | -                | +               | +                    |
| $x-\sqrt{3}$ | -                      | -                | -               | +                    |
| $x+\sqrt{3}$ | -                      | +                | +               | +                    |
|              | -                      | ⊕                | -               | ⊕                    |

$$\Rightarrow \text{ib sú } x_0=0 \wedge x_0=-\sqrt{3} \wedge x_0=\sqrt{3}$$

konvexná na  $(-\sqrt{3}; 0) \cup (0; \sqrt{3})$

konkávná na  $(-\infty; -\sqrt{3}) \cup (0; \sqrt{3})$

(6.1) mu)  $f: y = \frac{x}{1-x^2}$  ;  $D(f) = (-\infty; -1) \cup (-1; 1) \cup (1; \infty) = \mathbb{R} \setminus \{-1; 1\}$

$$y' = \frac{1 \cdot (1-x^2) - x \cdot (-2x)}{(1-x^2)^2} = \frac{1-x^2 + 2x^2}{(1-x^2)^2} = \frac{x^2+1}{(1-x^2)^2}$$

$$y'' = \frac{2x \cdot (1-x^2)^2 - (x^2+1) \cdot 2 \cdot (1-x^2) \cdot (-2x)}{(1-x^2)^4} = \frac{(1-x^2) \cdot [2x(1-x^2) + 4x(x^2+1)]}{(1-x^2)^4} = \frac{2x - 2x^3 + 4x^3 + 4x}{(1-x^2)^3} =$$

$$= \frac{2x^3 + 6x}{(1-x^2)^3} = \frac{2x(x^2+3)}{(1-x^2)^3} = \underbrace{\frac{2(x^2+3)}{(1-x^2)^2}}_{>0} \cdot \frac{x}{1-x^2} = 0 \Leftrightarrow x_o = 0$$

$$\frac{2(x^2+3)}{(1-x^2)^2} \cdot \frac{x}{1-x^2} > 0 \Leftrightarrow \frac{x}{(1-x^2)} > 0 \Leftrightarrow \frac{x}{(1-x)(1+x)} > 0$$

|       | $(-\infty; -1)$ | $(-1; 0)$ | $(0; 1)$ | $(1; \infty)$ |
|-------|-----------------|-----------|----------|---------------|
| $x$   | -               | -         | +        | +             |
| $1-x$ | +               | +         | +        | -             |
| $1+x$ | -               | +         | +        | +             |
|       | (+)             | -         | (+)      | -             |

$\Rightarrow$  ib je  $x_o = 0$

konvexná na  $(-\infty; -1)$  a na  $(0; 1)$

konkávnas na  $(-1; 0)$  a na  $(1; \infty)$

(60)

$$n) f: y = \frac{x^2}{16-x^2} ; D(f) = \mathbb{R} \setminus \{-4; 4\}$$

(60)

$$y' = \frac{2x(16-x^2) - x^2(-2x)}{(16-x^2)^2} = \frac{32x - 2x^3 + 2x^3}{(16-x^2)^2} = \frac{32x}{(16-x^2)^2}$$

$$y'' = \frac{32(16-x^2)^2 - 32x(16-x^2) \cdot 2 \cdot (-2x)}{(16-x^2)^4} = \frac{(16-x^2) \cdot [32 \cdot (16-x^2) + 32 \cdot 4x^2]}{(16-x^2)^4} = \frac{32 \cdot (16-x^2 + 4x^2)}{(16-x^2)^3} = \frac{32 \cdot (3x^2 + 16)}{(16-x^2)^3} =$$

$$= \frac{32 \cdot (3x^2 + 16)}{(16-x^2)^3} = 0 \Leftrightarrow 3x^2 + 16 = 0, \text{ also } 3x^2 + 16 > 0 \text{ stets}$$

$$\frac{32(3x^2+16)}{(16-x^2)^3} = \underbrace{\frac{32(3x^2+16)}{(16-x^2)^2}}_{>0} \cdot \frac{1}{16-x^2} > 0 \Leftrightarrow \frac{1}{16-x^2} > 0 \Leftrightarrow (4-x)(4+x) > 0$$

$$(4-x > 0 \wedge 4+x > 0) \vee (4-x < 0 \wedge 4+x < 0)$$

$$(x < 4 \wedge x > -4) \vee (x > 4 \wedge x < -4)$$

$$(-4; 4)$$

$\emptyset$

$\Rightarrow$  ib

konvex war  $(-4; 4)$

konkav war  $(-\infty; -4) \cup (4; \infty)$

(61)

6.2) a)  $f: y = \frac{x^2+1}{x^2-1}$  ;  $D(f) = \mathbb{R} \setminus \{-1; 1\}$

$$y' = \frac{2x(x^2-1) - (x^2+1) \cdot 2x}{(x^2-1)^2} = \frac{2x[x^2-1-x^2-1]}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

$$y'' = \frac{-4 \cdot (x^2-1)^2 - (-4x) \cdot 2(x^2-1) \cdot 2x}{(x^2-1)^4} = \frac{(x^2-1) \cdot [-4(x^2-1) + 16x^2]}{(x^2-1)^4} = \frac{-4x^2+4+16x^2}{(x^2-1)^3} = \frac{12x^2+4}{(x^2-1)^3} = 0 \Leftrightarrow 12x^2+4=0, \text{ ale } 12x^2+4>0 \text{ vždy}$$

$$\frac{12x^2+4}{(x^2-1)^3} > 0 \Leftrightarrow \underbrace{\frac{12x^2+4}{(x^2-1)^2}}_{>0} \cdot \frac{1}{x^2-1} > 0 \Leftrightarrow \frac{1}{x^2-1} > 0 \Leftrightarrow x^2-1 > 0 \Leftrightarrow (x-1)(x+1) > 0$$

$(x > 1 \wedge x > -1) \vee (x < 1 \wedge x < -1)$   
 $(1; \infty) \quad \cup \quad (-\infty; -1)$

$\Rightarrow$  ib  $\nexists$   
 konvexná na  $(-\infty; -1)$  a na  $(1; \infty)$   
 konkávná na  $(-1; 1)$

b)  $f: y = \frac{1}{x^3} + \frac{1}{x^2}$  ;  $D(f) = \mathbb{R} \setminus \{0\}$

$$y''' = \left(\frac{-3}{x^4} + \frac{-2}{x^3}\right)' = \left(-3x^{-4} - 2x^{-3}\right)' = 12x^{-5} + 6x^{-4} = \frac{12}{x^5} + \frac{6}{x^4} = \frac{12+6x}{x^5} = 0 \Leftrightarrow 12+6x=0 \Leftrightarrow x_0 = -2$$

$$\frac{12+6x}{x^5} > 0 \Leftrightarrow \underbrace{\frac{1}{x^4}}_{>0} \cdot \frac{12+6x}{x} > 0 \Leftrightarrow \frac{12+6x}{x} > 0 \Leftrightarrow \begin{cases} (12+6x > 0 \wedge x > 0) \vee (12+6x < 0 \wedge x < 0) \\ (x > -2 \wedge x > 0) \vee (x < -2 \wedge x < 0) \end{cases}$$

$(0; \infty) \quad \cup \quad (-\infty; -2)$

$\Rightarrow$  ib je  $x_0 = -2$   
 konvexná na  $(-\infty; -2)$  a na  $(0; \infty)$ ; konkávná na  $(-2; 0)$

(62)

c)  $f: y = \frac{1}{x^3} - \frac{6}{x}$  ;  $D(f) = \mathbb{R} \setminus \{0\}$

$$y'' = (x^{-3} - 6x^{-1})'' = (-3x^{-4} + 6x^{-2}) = 12x^{-5} - 12x^{-3} = \frac{12}{x^5} - \frac{12}{x^3} = \frac{12 - 12x^2}{x^5} = 0 \Leftrightarrow 12 - 12x^2 = 0 \Leftrightarrow x^2 = 1$$

$\boxed{x_0 = -1 \wedge x_0 = 1}$

$$\frac{12 - 12x^2}{x^5} > 0 \Leftrightarrow \underbrace{\frac{12}{x^4}}_{> 0} \cdot \frac{1-x^2}{x} > 0 \Leftrightarrow \frac{(1-x)(1+x)}{x} > 0$$

|       | $(-\infty; -1)$ | $(-1; 0)$ | $(0; 1)$ | $(1; \infty)$ |
|-------|-----------------|-----------|----------|---------------|
| $x$   | -               | -         | +        | +             |
| $1-x$ | +               | +         | +        | -             |
| $1+x$ | -               | +         | +        | +             |
|       | $\oplus$        | -         | $\oplus$ | -             |

$\Rightarrow$  ib je  $x_0 = 1 \wedge x_0 = -1$

Konvexná na

$(-\infty; -1)$  a na  $(0; 1)$

Konkávná na

$(-1; 0)$  a na  $(1; \infty)$

d)  $f: y = \left(\frac{1}{2} + \frac{1}{x}\right)^2$  ;  $D(f) = \mathbb{R} \setminus \{0\}$

$$y' = \left[ \left( \frac{x+2}{2x} \right)^2 \right]' = 2 \cdot \frac{x+2}{2x} \cdot \frac{1 \cdot 2x - (x+2) \cdot 2}{(2x)^2} = \frac{2(x+2)(2x-2x-4)}{(2x)^3} = \frac{-8(x+2)}{(2x)^3} = \frac{-8x-16}{(2x)^3}$$

$$y'' = \frac{-8(2x)^3 - (-8x-16) \cdot 3 \cdot (2x)^2 \cdot 2}{(2x)^6} = \frac{(2x)^2 \cdot [-8 \cdot 2x + 6 \cdot (8x+16)]}{(2x)^6} = \frac{-16x + 48x + 96}{(2x)^4} = \frac{32x + 96}{(2x)^4} = 0 \Leftrightarrow 32x + 96 = 0 \Leftrightarrow x = -3$$

$$\frac{32x + 96}{(2x)^4} = \frac{32}{(2x)^4} \cdot \frac{x+3}{1} > 0 \Leftrightarrow x+3 > 0 \Leftrightarrow \boxed{x > -3} \Rightarrow$$

ib je  $x_0 = -3$

Konvexná na  $(-3; 0)$  a na  $(0; \infty)$

Konkávná na  $(-\infty; -3)$

(6.2)

e)  $f: y = 3x - \sqrt{x-3}$ ;  $D(f): x-3 \geq 0$   
 $x \geq 3 \Rightarrow D(f) = [3; \infty)$

$$y' = \left(3 - \frac{1}{2}(x-3)^{-\frac{1}{2}}\right)' = \frac{1}{2}(x-3)^{-\frac{3}{2}} = \frac{1}{4\sqrt{(x-3)^3}} > 0 \text{ vždy} \Rightarrow$$

~~je ib~~  
konvexná na  $[3; \infty)$

f)  $f: y = 4 + \sqrt[3]{x^2}$ ; ~~(D(f) = \mathbb{R})~~  $D(f) = \mathbb{R}$

$$y' = \left(\frac{2}{3}x^{-\frac{1}{3}}\right)' = -\frac{2}{9}x^{-\frac{4}{3}} = \frac{-2}{9\sqrt[3]{x^4}} \rightsquigarrow$$

čítať  $< 0$  vždy }  
 menovateľ  $> 0$  vždy }  $\Rightarrow \frac{-2}{9\sqrt[3]{x^4}} < 0$  vždy  $\Rightarrow$

~~je ib~~  
konkávná na  $(-\infty; \infty)$

g)  $f: y = \frac{2x}{\sqrt{x^2+1}}$ ;  $D(f) = \mathbb{R}$

$$y'' = \frac{2 \cdot (x^2+1)^{\frac{1}{2}} - 2x \cdot \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot 2x}{(\sqrt{x^2+1})^2} = \frac{2\sqrt{x^2+1} - 2x \cdot \frac{1}{\sqrt{x^2+1}}}{x^2+1} = \frac{\frac{2(x^2+1) - 2x^2}{\sqrt{x^2+1}}}{x^2+1} = \frac{2x^2+2-2x^2}{(x^2+1) \cdot \sqrt{x^2+1}} = \frac{2}{(x^2+1) \cdot (x^2+1)^{\frac{1}{2}}} =$$

$$= \frac{2}{(x^2+1)^{\frac{3}{2}}}$$

$$y''' = \frac{-2 \cdot \frac{3}{2} \cdot (x^2+1)^{\frac{1}{2}} \cdot 2x}{[(x^2+1)^{\frac{3}{2}}]^2} = \frac{-6x \cdot (x^2+1)^{\frac{1}{2}}}{(x^2+1)^3} = \frac{-6x}{(x^2+1)^{\frac{5}{2}}} = \frac{-6}{(x^2+1)^{\frac{1}{2}}} \cdot \frac{x}{(x^2+1)^{\frac{1}{2}}} = \frac{-6}{(x^2+1)^{\frac{1}{2}} \cdot \sqrt{x^2+1}} \cdot x = 0 \Leftrightarrow x = 0$$

$$\frac{-6}{(x^2+1)^{\frac{1}{2}} \cdot \sqrt{x^2+1}} \cdot x > 0 \Leftrightarrow x < 0, \text{ lebo } [\text{záporné č.}] \cdot x > 0 \Leftrightarrow x < 0 \Rightarrow$$

ib je  $x_0 = 0$   
 na  $(-\infty; 0)$  je konvexná  
 na  $(0; \infty)$  je konkávná

(6.3)

$$a^x \cdot b^x = (a \cdot b)^x$$

$$\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$$

$$(a^x)^y = a^{x \cdot y}$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

(62)

$$h) f: y = \frac{x}{\sqrt{x^3+1}} ; D(f) = \underbrace{x^3+1 \geq 0}_{x^3 > 0} \wedge \underbrace{\sqrt{x^3+1} \neq 0}_{x^3 > -1} \Leftrightarrow x > -1 \Rightarrow D(f) = (-1; \infty)$$

(64)

$$\begin{aligned} y' &= \frac{1 \cdot \frac{1}{2}(x^3+1)^{-\frac{1}{2}} - x \cdot \frac{1}{2}(x^3+1)^{-\frac{1}{2}} \cdot 3x^2}{x^3+1} = \frac{\frac{1}{2}(x^3+1)^{-\frac{1}{2}} - \frac{3}{2}x^3(x^3+1)^{-\frac{1}{2}}}{x^3+1} = \frac{\frac{1}{2}(x^3+1)^{-\frac{1}{2}} - \frac{3}{2}x^3(x^3+1)^{-\frac{1}{2}}}{x^3+1} = \frac{-x^3 + \frac{1}{2}}{(x^3+1)^{\frac{3}{2}}} \\ y'' &= \frac{(-3x^2)(x^3+1)^{\frac{3}{2}} - (-x^3+1) \cdot \frac{3}{2} \cdot (x^3+1)^{-\frac{1}{2}} \cdot 3x^2}{[(x^3+1)^{\frac{3}{2}}]^2} = \frac{(x^3+1)^{\frac{1}{2}} \cdot [-3x^2 \cdot (x^3+1) - \frac{9}{2}x^2 \cdot (-x^3 + \frac{1}{2})]}{(x^3+1)^{\frac{5}{2}}} = \frac{-3x^5 - 3x^2 + \frac{9}{2}x^5 - \frac{9}{4}x^2}{(x^3+1)^{\frac{5}{2}}} = \\ &= \frac{x^5 \cdot (-\frac{6}{2} + \frac{9}{2}) + x^2 \cdot (-\frac{12}{4} - \frac{9}{4})}{(x^3+1)^{\frac{5}{2}}} = \frac{\frac{3}{2}x^5 - \frac{21}{4}x^2}{(x^3+1)^{\frac{5}{2}}} = \frac{\frac{3}{4}x^2 \cdot (2x^3 - 7)}{(x^3+1)^{\frac{2}{2}} \cdot \sqrt{x^3+1}} = \frac{\frac{3}{4}x^2}{(x^3+1)^{\frac{2}{2}} \cdot \sqrt{x^3+1}} \cdot (2x^3 - 7) = 0 \Leftrightarrow \\ &\quad v_0 \quad v_0 \end{aligned}$$

$$\Leftrightarrow x^2 = 0 \quad v \quad 2x^3 - 7 = 0$$

$$\underline{x_0 = 0} \quad v \quad x^3 = \frac{7}{2}$$

$$\underline{x_0 = \sqrt[3]{\frac{7}{2}}}$$

$$\begin{aligned} y'' > 0 &\Leftrightarrow 2x^3 - 7 > 0 \\ x^3 &> \frac{7}{2} \\ \underline{x > \sqrt[3]{\frac{7}{2}}} \end{aligned}$$

$$\Rightarrow \text{ib sú } x_0 = 0 \wedge x_0 = \sqrt[3]{\frac{7}{2}}$$

könvergencia műv  $(\sqrt[3]{\frac{7}{2}}; \infty)$

konkáv műv  $(-\infty; 0) \cup (0; \sqrt[3]{\frac{7}{2}})$

(6.3)

$$a) f: y = x \cdot e^{-x}$$

$$y'' = (x \cdot e^{-x})'' = [1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1)]' = [e^{-x} \cdot (1-x)]' = e^{-x} \cdot (-1) \cdot (1-x) + e^{-x} \cdot (-1) = e^{-x} \cdot (-1 \cdot (1-x) - 1) = e^{-x} \cdot (-1+x-1) = e^{-x} \cdot (x-2) = 0 \Leftrightarrow x_o = 2$$

$$y'' > 0 \Leftrightarrow \frac{x-2}{e^{-x}} > 0 \Leftrightarrow x-2 > 0 \Leftrightarrow x > 2 \Rightarrow$$

ib je  $x_o = 2$ konvexná na  $(2; \infty)$ konkávná na  $(-\infty; 2)$ 

$$b) f: y = e^{-x^2}$$

$$y'' = [e^{-x^2} \cdot (-2x)]' = e^{-x^2} \cdot (-2x) \cdot (-2x) + e^{-x^2} \cdot (-2) = e^{-x^2} \cdot ((-2x)^2 - 2) = \frac{4x^2 - 2}{e^{x^2}} = 0 \Leftrightarrow 4x^2 - 2 = 0 \Leftrightarrow x^2 = \frac{1}{2} \Leftrightarrow x_o = \pm \sqrt{\frac{1}{2}}$$

$$y'' > 0 \Leftrightarrow \frac{4x^2 - 2}{e^{x^2}} > 0 \Leftrightarrow 4x^2 - 2 > 0$$

$$4x^2 > 2$$

$$x^2 > \frac{1}{2} \Rightarrow (-\infty; -\sqrt{\frac{1}{2}}) \cup (\sqrt{\frac{1}{2}}; \infty)$$

$$(x - \sqrt{\frac{1}{2}})(x + \sqrt{\frac{1}{2}}) > 0$$

---

ib je  $x_o = -\sqrt{\frac{1}{2}}$  a  $x_o = \sqrt{\frac{1}{2}}$ konvexná na  $(-\infty; -\sqrt{\frac{1}{2}})$  a na  $(\sqrt{\frac{1}{2}}; \infty)$ konkávná na  $(-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}})$ 

$$c) f: y = x \cdot e^{-x^2}$$

$$y'' = 1 \cdot e^{-x^2} + x \cdot e^{-x^2} \cdot (-2x) = e^{-x^2} \cdot (1 - 2x^2)$$

$$y'' = e^{-x^2} \cdot (-2x) \cdot (1 - 2x^2) + e^{-x^2} \cdot (-4x) = e^{-x^2} \cdot (-2x \cdot (1 - 2x^2) - 4x) = e^{-x^2} \cdot (-2x + 4x^3 - 4x) = e^{-x^2} \cdot (4x^3 - 6x) = \frac{4x^3 - 6x}{e^{x^2}} = 0 \Leftrightarrow$$

$$\Leftrightarrow 4x^3 - 6x = 0 \Leftrightarrow 2x(2x^2 - 3) = 0 \Leftrightarrow x_o = 0 \vee$$

$$\vee 2x^2 - 3 = 0 \Leftrightarrow x_o = \pm \sqrt{\frac{3}{2}}$$

(63)

$$c) y'' > 0 \Leftrightarrow \frac{4x^3 - 6x}{e^x} > 0 \Leftrightarrow 4x^3 - 6x > 0 \Leftrightarrow 2x(2x^2 - 3) > 0 \Leftrightarrow 4x(x^2 - \frac{3}{2}) > 0 \Leftrightarrow x \cdot (x - \sqrt{\frac{3}{2}}) \cdot (x + \sqrt{\frac{3}{2}}) > 0$$

(66)

ib sú  $x_0 = 0$   
 $x_0 = -\sqrt{\frac{3}{2}}$   
 $x_0 = \sqrt{\frac{3}{2}}$

konvexná na  $(-\sqrt{\frac{3}{2}}, 0)$   
a na  $(\sqrt{\frac{3}{2}}, \infty)$

konkávná na  $(-\infty, -\sqrt{\frac{3}{2}})$   
a na  $(0, \sqrt{\frac{3}{2}})$

|                          | $(-\infty, -\sqrt{\frac{3}{2}})$ | $(-\sqrt{\frac{3}{2}}, 0)$ | $(0, \sqrt{\frac{3}{2}})$ | $(\sqrt{\frac{3}{2}}, \infty)$ |
|--------------------------|----------------------------------|----------------------------|---------------------------|--------------------------------|
| $x$                      | -                                | -                          | +                         | +                              |
| $x - \sqrt{\frac{3}{2}}$ | -                                | -                          | -                         | +                              |
| $x + \sqrt{\frac{3}{2}}$ | -                                | +                          | +                         | +                              |
|                          | -                                | ⊕                          | -                         | ⊕                              |

d)  $f: y = x^2 e^{-x}$

$$y'' = 2x \cdot e^{-x} + x^2 \cdot e^{-x} \cdot (-1) = e^{-x} \cdot (2x - x^2)$$

$$y'' = e^{-x} \cdot (-1) \cdot (2x - x^2) + e^{-x} \cdot (2 - 2x) = e^{-x} \cdot (x^2 - 4x + 2) = \frac{x^2 - 4x + 2}{e^x} = 0 \Leftrightarrow x^2 - 4x + 2 = 0$$

$$y'' > 0 \Leftrightarrow \frac{x^2 - 4x + 2}{e^x} > 0 \Leftrightarrow (x - 2 + \sqrt{2})(x - 2 - \sqrt{2}) > 0$$

$$(x > 2 - \sqrt{2} \wedge x > 2 + \sqrt{2}) \vee (x < 2 - \sqrt{2} \wedge x < 2 + \sqrt{2})$$

$$(2 + \sqrt{2}; \infty) \quad \cup \quad (-\infty; 2 - \sqrt{2})$$

$\Rightarrow$  ib sú  $x_0 = 2 - \sqrt{2}$  a  $x_0 = 2 + \sqrt{2}$

konvexná na  $(-\infty, 2 - \sqrt{2})$  a na  $(2 + \sqrt{2}, \infty)$

konkávná na  $(2 - \sqrt{2}, 2 + \sqrt{2})$

$$x_{1,2} = \frac{4 \pm \sqrt{16 - 8}}{2} = \frac{4 \pm \sqrt{4 \cdot 2}}{2} = \frac{4 \pm 2 \cdot \sqrt{2}}{2}$$

$$= \frac{2 \cdot (2 \pm \sqrt{2})}{2} = 2 \pm \sqrt{2}$$

$$\Rightarrow x_0 = 2 - \sqrt{2} \quad \text{a} \quad x_0 = 2 + \sqrt{2}$$

(6.3)

$$e) f: y = x \cdot e^{\frac{1}{x}}; D(f) = \mathbb{R} \setminus \{0\}$$

$$y'' = [1 \cdot e^{\frac{1}{x}} + x \cdot e^{\frac{1}{x}} \cdot (-x^{-2})]' = \left[ e^{\frac{1}{x}} \cdot (1 - x^{-1}) \right]' = e^{\frac{1}{x}} \cdot (-x^{-2}) \cdot (1 - x^{-1}) + e^{\frac{1}{x}} \cdot x^{-2} = e^{\frac{1}{x}} \left( \frac{1 - \frac{1}{x}}{-x^2} + \frac{1}{x^2} \right) = e^{\frac{1}{x}} \cdot \frac{\frac{1}{x} - 1 + 1}{x^2} =$$

$$= e^{\frac{1}{x}} \cdot \frac{1}{x^3} = \frac{e^{\frac{1}{x}} > 0}{x^3} \neq 0 \text{ nukolý}$$

ib

$$y'' > 0 \Leftrightarrow \frac{e^{\frac{1}{x}}}{x^3} > 0 \Leftrightarrow x^3 > 0 \Leftrightarrow x > 0 \Rightarrow \begin{array}{l} \text{konvexná na } (0; \infty) \\ \text{konkávná na } (-\infty; 0) \end{array}$$

$$f) f: y = e^{4 - \frac{x^2}{2}}$$

$$y'' = \left[ e^{4 - \frac{x^2}{2}} \cdot \left( -\frac{1}{2} \cdot 2x \right) \right]' = \left( -x \cdot e^{4 - \frac{x^2}{2}} \right)' = -1 \cdot e^{4 - \frac{x^2}{2}} + (-x) \cdot e^{4 - \frac{x^2}{2}} \cdot (-x) = e^{4 - \frac{x^2}{2}} \cdot (-1 + x^2) = (x^2 - 1) \cdot e^{4 - \frac{x^2}{2}} = 0 \Leftrightarrow x^2 = 1 \Rightarrow \underline{x_0 = -1 \wedge x_0 = 1}$$

$$y'' > 0 \Leftrightarrow (x^2 - 1) \cdot e^{4 - \frac{x^2}{2}} > 0 \Leftrightarrow x^2 - 1 > 0 \Leftrightarrow x > 1 \Rightarrow (-\infty; -1) \cup (1; \infty) \Rightarrow$$

$$(x+1)(x-1) > 0$$

ib sú  $x_0 = -1$  a  $x_0 = 1$ konvexná na  $(-\infty; -1)$  a na  $(1; \infty)$ konkávná na  $(-1; 1)$ 

$$g) f: y = e^{1 - \frac{x^3}{3}}$$

$$y''' = \left[ e^{1 - \frac{x^3}{3}} \cdot \left( -\frac{1}{3} \cdot 3x^2 \right) \right]' = \left( e^{1 - \frac{x^3}{3}} \cdot (-x^2) \right)' = e^{1 - \frac{x^3}{3}} \cdot (-x^2) \cdot (-x^2) + e^{1 - \frac{x^3}{3}} \cdot (-2x) = e^{1 - \frac{x^3}{3}} \cdot (x^4 - 2x) = x \cdot e^{1 - \frac{x^3}{3}} \cdot (x^3 - 2) = 0 \Leftrightarrow$$

$$x e^{1 - \frac{x^3}{3}} \cdot (x^3 - 2) > 0 \Leftrightarrow x(x^3 - 2) > 0 \Leftrightarrow (x > 0 \wedge x > \sqrt[3]{2}) \vee (x < 0 \wedge x < -\sqrt[3]{2})$$

$$(\sqrt[3]{2}; \infty) \cup (-\infty; 0)$$

$$\underline{x_0 = \sqrt[3]{2}}$$

(6.7)

g) ib sú  $x_0=0$  a  $x_0=\sqrt[3]{2}$

konvexná na  $(-\infty; 0)$  a na  $(\sqrt[3]{2}; \infty)$

konkávná na  $(0; \sqrt[3]{2})$

h)  $f: y = e^{2x} - 8e^x + 5$

$$y'' = [e^{2x} \cdot 2 - 8e^x] = 4e^{2x} - 8e^x = \underbrace{4e^x}_{>0} \cdot (e^x - 2) = 0 \Leftrightarrow e^x = 2 \Leftrightarrow x = \ln 2$$

$$4e^x(e^x - 2) > 0 \Leftrightarrow e^x > 2 \Leftrightarrow x > \ln 2 \Rightarrow \text{ib je } x_0 = \ln 2$$

konvexná na  $(\ln 2; \infty)$

konkávná na  $(-\infty; \ln 2)$

i)  $f: y = (2-x^2) \cdot e^{-x}$

$$y'' = [-2x \cdot e^{-x} + (2-x^2) \cdot e^{-x} \cdot (-1)] = [e^{-x} \cdot (-2x-2+x^2)] = [e^{-x}(x^2-2x-2)] = e^{-x} \cdot (-1) \cdot (x^2-2x-2) + e^{-x} \cdot (2x-2) = e^{-x} \cdot (-x^2+2x+2+2x-2) = e^{-x} \cdot (-x^2+4x) = x e^{-x}(4-x) = 0 \Leftrightarrow x_0 = 0 \wedge x_0 = 4$$

$$y'' > 0 \Leftrightarrow x(4-x)e^{-x} > 0 \Leftrightarrow (x > 0 \wedge x < 4) \vee (x < 0 \wedge x > 4) \Rightarrow (0; 4)$$

ib sú  $x_0=0$  a  $x_0=4$

konvexná na  $(0; 4)$

konkávná na  $(-\infty; 0)$  a na  $(4; \infty)$

(68) j)  $f: y = \frac{e^x}{x}$  ;  $D(f) = \mathbb{R} \setminus \{0\}$

$$y'' = \left( \frac{e^x \cdot x - e^x \cdot 1}{x^2} \right)' = \left( \frac{e^x \cdot (x-1)}{x^2} \right)' = \frac{[e^x \cdot (x-1) + e^x \cdot 1] \cdot x^2 - e^x \cdot (x-1) \cdot 2x}{x^4} = \frac{x^2 \cdot e^x \cdot (x-1+1) - 2x \cdot e^x \cdot (x-1)}{x^4} = \frac{x e^x \cdot (x^2 - 2(x-1))}{x^4} =$$

$$= \frac{e^x \cdot (x^2 - 2x + 2)}{x^3} = 0 \Leftrightarrow x^2 - 2x + 2 = 0$$

$$D = 4 - 4 \cdot 2 < 0 \rightarrow \text{obsadíme } 0: 0^2 - 2 \cdot 0 + 2 = 2 > 0 \Rightarrow x^2 - 2x + 2 > 0 \text{ vždy } (\forall x \in D(f))$$

$$y'' > 0 \Leftrightarrow \frac{\overset{0}{e^x}(\overset{x^2-2x+2}{\cancel{x^3}})}{\cancel{x^3}} > 0 \Leftrightarrow x^3 > 0 \Leftrightarrow x > 0 \Rightarrow$$

ib  $\nexists$   
konvexná na  $(0; \infty)$   
konkávná na  $(-\infty; 0)$

L'Hospitalovo pravidlo:

Ak  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{0}{0}$  alebo  $\frac{\pm \infty}{\pm \infty}$ , potom

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

(69)

7.1

$$\lim_{x \rightarrow 1} \frac{x^3 - 2x^2 + x}{x^2 - 1} = \frac{1^3 - 2 \cdot 1^2 + 1}{1^2 - 1} = \frac{0}{0}; \text{ môžeme teda použiť L'Hospitalovo pravidlo}$$

[ v ďalších príkladoch nebudem zistovať, či ide o limitu s výsledkom  $\frac{0}{0}$ , ale budem podľa zadania príkladov inu počítať limity použitím L'H pravidla ]

$$\lim_{x \rightarrow 1} \frac{x^3 - 2x^2 + x}{x^2 - 1} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 1} \frac{(x^3 - 2x^2 + x)'}{(x^2 - 1)'} = \lim_{x \rightarrow 1} \frac{3x^2 - 4x + 1}{2x} = \frac{3 \cdot 1^2 - 4 \cdot 1 + 1}{2 \cdot 1} = \frac{0}{2} = 0$$

$$\text{b)} \lim_{x \rightarrow 3} \frac{x^3 - 9x}{x^4 - 3x^3 - x + 3} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 3} \frac{3x^2 - 9}{4x^3 - 9x^2 - 1} = \lim_{x \rightarrow 3} \frac{3 \cdot 3^2 - 9}{4 \cdot 3^3 - 9 \cdot 3^2 - 1} = \frac{27 - 9}{108 - 81 - 1} = \frac{18}{26} = \frac{9}{13}$$

$$\text{c)} \lim_{x \rightarrow -1} \frac{x^4 + x^3 - 2x^2 - 3x - 1}{x^4 + 4x^2 - 5} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow -1} \frac{4x^3 + 3x^2 - 4x - 3}{4x^3 + 8x} = \frac{4 \cdot (-1)^3 + 3 \cdot (-1)^2 - 4 \cdot (-1) - 3}{4 \cdot (-1)^3 + 8 \cdot (-1)} = \frac{-4 + 3 + 4 - 3}{-4 - 8} = \frac{0}{-12} = 0$$

$$\text{d)} \lim_{x \rightarrow 1} \frac{\sqrt[3]{8x} - 2x}{\sqrt[4]{x} - x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 1} \frac{[(8x)^{\frac{1}{3}} - 2x]}{(x^{\frac{1}{4}} - x)} = \lim_{x \rightarrow 1} \frac{\frac{1}{3} \cdot (8x)^{\frac{-2}{3}} \cdot 8 - 2}{\frac{1}{4} x^{\frac{-3}{4}} - 1} = \lim_{x \rightarrow 1} \frac{\frac{8}{3 \cdot \sqrt[3]{(8x)^2}} - 2}{\frac{1}{4} \cdot \sqrt[4]{x^3} - 1} = \frac{\frac{2}{3} - 2}{\frac{1}{4} - 1} = \frac{-\frac{4}{3}}{-\frac{3}{4}} = \frac{16}{9}$$

$a^{\frac{x}{4}} = \sqrt[4]{a^x}$

$$\text{e)} \lim_{x \rightarrow 1} \frac{\sqrt{x+1} - \sqrt{2}}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{(x+1)^{\frac{1}{2}} - \sqrt{2}}{x^2 - 1} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 1} \frac{\frac{1}{2}(x+1)^{-\frac{1}{2}}}{2x} = \lim_{x \rightarrow 1} \frac{\frac{1}{2\sqrt{x+1}}}{2x} = \lim_{x \rightarrow 1} \frac{1}{4x\sqrt{x+1}} = \frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8}$$

(splatíme sa v menovateli nepísati  $\sqrt{m^n}$ )

7.0

$$\text{f)} \lim_{x \rightarrow 0} \frac{5^x - 1}{x} \stackrel{\text{LH}}{=} \lim_{x \rightarrow 0} \frac{5^x \cdot \ln 5}{1} = \frac{5^0 \cdot \ln 5}{1} = \underline{\underline{\ln 5}}$$

$$\text{g)} \lim_{x \rightarrow 1} \frac{3^x - 3}{3^x - 3} \stackrel{\text{LH}}{=} \lim_{x \rightarrow 1} \frac{9x^2}{3^x \cdot \ln 3} = \frac{9 \cdot 1^2}{3^1 \cdot \ln 3} = \frac{9}{3 \ln 3} = \underline{\underline{\frac{3}{\ln 3}}}$$

$$\text{h)} \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x} \stackrel{\text{LH}}{=} \lim_{x \rightarrow 0} \frac{e^x + e^{-x}}{1} = \underline{\underline{e^0 + e^0}} = \underline{\underline{2}}$$

$$\text{i)} \lim_{x \rightarrow 0} \frac{e^x - 1}{x^2} \stackrel{\text{LH}}{=} \lim_{x \rightarrow 0} \frac{e^x}{2x} = \frac{1}{0} \quad \underline{\underline{\infty}}$$

$$\text{j)} \lim_{x \rightarrow \frac{\pi}{3}} \frac{1 - 2 \cos x}{\pi - 3x} \stackrel{\text{LH}}{=} \lim_{x \rightarrow \frac{\pi}{3}} \frac{-2 \cdot (-\sin x)}{-3} = \frac{2}{3} \cdot \left(-\sin \frac{\pi}{3}\right) = -\frac{2}{3} \cdot \frac{\sqrt{3}}{2} = \underline{\underline{-\frac{\sqrt{3}}{3}}}$$

$$\text{k)} \lim_{x \rightarrow 2} \frac{3 \log(\pi x)}{2-x} \stackrel{\text{LH}}{=} \lim_{x \rightarrow 2} \frac{3 \cdot \frac{1}{\cos^2(\pi x)} \cdot \pi}{-1} = \frac{3\pi}{-\cos^2(\pi \cdot 2)} = \frac{3\pi}{-1} = \underline{\underline{-3\pi}}$$

$$\text{l)} \lim_{x \rightarrow 0} \frac{\sin x}{e^x - 1} \stackrel{\text{LH}}{=} \lim_{x \rightarrow 0} \frac{\cos x}{e^x} = \frac{\cos 0}{e^0} = \frac{1}{1} = \underline{\underline{1}}$$

$$\text{m)} \lim_{x \rightarrow \frac{\pi}{2}} \frac{\arctg\left(x - \frac{\pi}{2}\right)}{\pi - 2x} \stackrel{\text{LH}}{=} \lim_{x \rightarrow \frac{\pi}{2}} \frac{\frac{1}{1+(x-\frac{\pi}{2})^2}}{-2} = \frac{1}{-2 \cdot (1+0^2)} = \underline{\underline{-\frac{1}{2}}}$$

$$\text{n)} \lim_{x \rightarrow 0} \frac{\arctg 3x}{\arcsin 2x} \stackrel{\text{LH}}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{1+(3x)^2} \cdot 3}{\frac{1}{\sqrt{1-(2x)^2}} \cdot 2} = \frac{3}{2} \cdot \frac{\sqrt{1-(2 \cdot 0)^2}}{1+(3 \cdot 0)^2} = \frac{3}{2} \cdot \frac{1}{1} = \underline{\underline{\frac{3}{2}}}$$

(72)

$$\text{a) } \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{2x - \sin x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{e^x + e^{-x}}{2 - \cos x} = \frac{e^0 + e^0}{2 - \cos 0} = \frac{1+1}{2-1} = \frac{2}{1} = \underline{\underline{2}}$$

$$\text{b) } \lim_{x \rightarrow 0} \frac{x^3 + \pi x}{\sin 3x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{3x^2 + \pi}{\cos 3x \cdot 3} = \frac{3 \cdot 0^2 + \pi}{\cos 0 \cdot 3} = \frac{\pi}{3}$$

$$\text{c) } \lim_{x \rightarrow 0} \frac{\ln(1+4x)}{3^x - 1} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{1+4x} \cdot 4}{3^x \cdot \ln 3} = \frac{\frac{4}{1+4 \cdot 0}}{3^0 \cdot \ln 3} = \frac{4}{1 \cdot 1 \cdot \ln 3} = \frac{4}{\ln 3}$$

$$\text{d) } \lim_{x \rightarrow 0} \frac{\sin 4x}{\ln(1+\sin x)} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\cos 4x \cdot 4}{\frac{1}{1+\sin x} \cdot \cos x} = \frac{4 \cdot \cos(4 \cdot 0)}{\cos 0} = \frac{4}{1} = \underline{\underline{4}}$$

$$\text{e) } \lim_{x \rightarrow 0} \frac{3 \ln(1-2x)}{2 \operatorname{arctg} 3x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{3 \cdot \frac{1}{1-2x} \cdot (-2)}{2 \cdot \frac{1}{1+(3x)^2} \cdot 3} = \frac{\frac{-6}{1-2 \cdot 0}}{\frac{6}{1+(3 \cdot 0)^2}} = \frac{-6 \cdot 1}{1 \cdot 6} = \underline{\underline{-1}}$$

$$\text{f) } \lim_{x \rightarrow 0} \frac{\operatorname{arctg} x + x^2}{2^{3x} - 3^{2x}} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{1+x^2} + 2x}{2^{3x} \cdot \ln 2 \cdot 3 - 3^{2x} \cdot \ln 3 \cdot 2} = \lim_{x \rightarrow 0} \frac{\frac{1+2x \cdot (1+x^2)}{1+x^2}}{3 \cdot \ln 2 \cdot 2^{3x} - 2 \cdot \ln 3 \cdot 3^{2x}} = \frac{1+2 \cdot 0 \cdot (1+0^2)}{(1+0^2)(3 \cdot \ln 2 \cdot 2^0 - 2 \cdot \ln 3 \cdot 3^0)} =$$

$$= \frac{1}{3 \ln 2 - 2 \ln 3}$$

$$\text{g) } \lim_{x \rightarrow 0} \frac{e^{3x} - e^{-2x}}{2 \operatorname{arcsin} x - \sin x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\frac{e^{3x} \cdot 3 + e^{-2x} \cdot (-2)}{2 \sqrt{1-x^2}}}{-\cos x} = \frac{\frac{e^0 \cdot 3 + e^0 \cdot 2}{2 \sqrt{1-0^2}}}{-\cos 0} = \frac{5}{1} = \underline{\underline{5}}$$

$$\text{h) } \lim_{x \rightarrow 0} \frac{\ln(\cos 3x)}{\operatorname{arctg} 4x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\frac{1}{\cos 3x} \cdot (-\sin 3x) \cdot 3}{\frac{1}{1+(4x)^2} \cdot 4} = \lim_{x \rightarrow 0} \frac{\frac{-3 \sin 3x}{\cos 3x}}{\frac{4}{1+(4x)^2}} = \frac{\frac{-3 \cdot \sin 0 \cdot (1+0^2)}{4 \cdot \cos 0}}{\frac{4}{1+(4 \cdot 0)^2}} = \frac{0}{4} = \underline{\underline{0}}$$

(73)

$$\textcircled{7.2} \text{ i) } \lim_{x \rightarrow 0} \frac{(1+x)^2 - (1+2x)}{x^2 + 4x^3} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{(x^2 + 2x + 1 - 1 - 2x)}{(x^2 + 4x^3)} = \lim_{x \rightarrow 0} \frac{(x^2)}{(x^2 + 4x^3)} = \lim_{x \rightarrow 0} \frac{2x}{2x + 12x^2} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{2}{2 + 24x} = \underline{\underline{1}}$$

→ L'H pravidlo můžeme použít  
ají následně v 1 příkladu

$$\text{ii) } \lim_{x \rightarrow -1} \frac{(x^2 + 3x + 2)^2}{x^3 - 3x - 2} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow -1} \frac{2 \cdot (x^2 + 3x + 2) \cdot (2x + 3)}{3x^2 - 3} = \lim_{x \rightarrow -1} \frac{2 \cdot (2x^3 + 6x^2 + 4x + 3x^2 + 9x + 6)}{3x^2 - 3}$$

$$= \lim_{x \rightarrow -1} \frac{2(2x^3 + 9x^2 + 13x + 6)}{3x^2 - 3} = \frac{0}{0} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow -1} \frac{(4x^3 + 18x^2 + 26x + 12)}{(3x^2 - 3)} = \lim_{x \rightarrow -1} \frac{12x^2 + 36x + 26}{6x} = \frac{2}{-6} = \underline{\underline{-\frac{1}{3}}}$$

$$\text{iii) } \lim_{x \rightarrow 0} \frac{1 - \cos^4 x}{4x^2} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{-4\cos^3 x \cdot (-\sin x)}{8x} = \lim_{x \rightarrow 0} \frac{4\sin x \cos^3 x}{8x} = \frac{0}{0} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\cos x \cdot \cos^3 x + \sin x \cdot 3\cos^2 x \cdot (-\sin x)}{2} =$$

$$= \frac{1 \cdot 1^3 - 0 \cdot 3 \cdot 1^2}{2} = \underline{\underline{\frac{1}{2}}}$$

$$\text{iv) } \lim_{x \rightarrow 0} \frac{1 - \cos x}{2x \sin x} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\sin x}{2 \sin x + 2x \cos x} = \frac{0}{0} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow 0} \frac{\cos x}{2 \cos x + 2 \cos x - 2x \sin x} = \frac{1}{2 \cdot 1 + 2 \cdot 1 - 2 \cdot 0 \cdot 0} = \underline{\underline{\frac{1}{4}}}$$

$$\text{v) } \lim_{x \rightarrow \frac{\pi}{6}} \frac{\ln(\sin 3x)}{(6x - \pi)^2} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \frac{\pi}{6}} \frac{\frac{1}{\sin 3x} \cdot \cos 3x \cdot 3}{2(6x - \pi) \cdot 6} = \lim_{x \rightarrow \frac{\pi}{6}} \frac{3 \cos 3x}{12(6x - \pi) \sin 3x} = \frac{0}{0} \stackrel{\text{L'H}}{=} \lim_{x \rightarrow \frac{\pi}{6}} \frac{(3 \cos 3x)}{[12(6x - \pi) \sin 3x]} =$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{-3 \sin 3x \cdot 3}{12 \cdot [6 \sin 3x + (6x - \pi) \cdot \cos 3x \cdot 3]} = \left( \begin{matrix} \text{L'H} \\ \frac{0}{0} \end{matrix} \right) \frac{-3 \cdot 1 \cdot 3}{12 \cdot (6 \cdot 1 + 0)} = \frac{-9}{72} = \underline{\underline{-\frac{1}{8}}}$$

(7.2)

$$m) \lim_{x \rightarrow 0} \frac{x - \sin x}{e^x - e^{-x} - 2x} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{1 - \cos x}{e^x + e^{-x} - 2} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{\sin x}{e^x - e^{-x}} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{\cos x}{e^x + e^{-x}} = \frac{1}{2}$$

(74)

$$o) \lim_{x \rightarrow 0} \frac{e^{3x} - 3x - 1}{\sin^2 x} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{3e^{3x} - 3}{2\sin x \cdot \cos x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{3 \cdot 3e^{3x}}{2(\cos^2 x - \sin^2 x)} = \frac{9}{2}$$

$$p) \lim_{x \rightarrow 0} \frac{1 - \cos 3x}{\sin^2 x} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{\sin 3x \cdot 3}{2 \sin x \cos x} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{3 \cos 3x \cdot 3}{2(\cos^2 x - \sin^2 x)} = \frac{9}{2}$$

$$q) \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{e^{x^2} - 1} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{\sin 2x \cdot 2}{e^{x^2} \cdot 2x} = \lim_{x \rightarrow 0} \frac{\sin 2x}{x \cdot e^{x^2}} = \frac{0}{0} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{\cos 2x \cdot 2}{e^{x^2} + x e^{x^2} \cdot 2x} = \lim_{x \rightarrow 0} \frac{2 \cos 2x}{e^{x^2}(1+2x^2)} = \frac{2}{1} = 2$$

$$r) \lim_{x \rightarrow 0} \frac{x^3}{x - \arctg x} \stackrel{LH}{=} \lim_{x \rightarrow 0} \frac{3x^2}{1 - \frac{1}{1+x^2}} = \lim_{x \rightarrow 0} \frac{3x^2}{\frac{1+x^2-1}{1+x^2}} = \lim_{x \rightarrow 0} \frac{3x^2(1+x^2)}{x^2} = \lim_{x \rightarrow 0} 3 \cdot (1+x^2) = 3$$

(8.1)

$$a) f: y = x^2 - 6x + 1$$

$y' = 2x - 6 = 0 \Rightarrow$  položime rovné 0 a zistíme,  
pie ktoré  $x$  platí rovnosť

$$2x = 6$$

$$\boxed{x_0 = 3}$$

$y'' = 2 > 0 \Rightarrow$  v  $x_0 = 3$  je stacionárny bod

a keďže 2. derivácia je  $> 0$ ,

v bode  $\boxed{x_0 = 3}$  má funkcia

lokálne minimum

(8.1)

$$b) f: y = 3 + 10x - 5x^2$$

$$y' = 10 - 10x = 0$$

$$\underline{x_0 = 1}$$

$$c) f: y = x^3 - 3x^2 - 9x + 7$$

$$y' = 3x^2 - 6x - 9 = 0$$

$$D = 36 + 4 \cdot 3 \cdot 9 = 144$$

$$x_{1,2} = \frac{6 \pm \sqrt{144}}{6} = \begin{cases} 3 \\ -1 \end{cases}$$

$$y'' = 6x - 6$$

$$y''(3) = 6 \cdot 3 - 6 = 12 > 0 \Rightarrow \forall x_0 = 3 \text{ má funkcia lok. minimum}$$

$$y''(-1) = 6 \cdot (-1) - 6 = -12 < 0 \Rightarrow \forall x_0 = -1 \text{ má funkcia lok. maximum}$$

$$d) f: y = x - \frac{16}{3}x^3$$

$$y' = 1 - \frac{16}{3} \cdot 3x^2 = 1 - 16x^2 = 0$$

$$16x^2 = 1$$

$$x^2 = \frac{1}{16}$$

$$x = \pm \frac{1}{4}$$

$$y'' = -16 \cdot 2x = -32x$$

$$y''\left(-\frac{1}{4}\right) = -16 \cdot 2 \cdot \left(-\frac{1}{4}\right) = 8 > 0 \Rightarrow \forall x_0 = -\frac{1}{4} \text{ lok. minimum}$$

$$y''\left(\frac{1}{4}\right) = -32 \cdot \frac{1}{4} = -8 < 0 \Rightarrow \forall x_0 = \frac{1}{4} \text{ lok. maximum}$$

$$e) f: y = (x+2)^2(x+5)$$

$$\begin{aligned} y' &= 2(x+2)(x+5) + (x+2)^2 = (x+2)(2x+10+x+2) = (x+2)(3x+12) = 0 \\ &= 3(x+2)(x+4) = 0 \Leftrightarrow x_0 = -2 \wedge x_0 = -4 \end{aligned}$$

$$y'' = 3(x+4) + 3(x+2) = 3x+12+3x+6 = 6x+18$$

$$y''(-2) = 6 \cdot (-2) + 18 = 6 > 0 \Rightarrow \forall x_0 = -2 \text{ lok. minimum}$$

$$y''(-4) = 6 \cdot (-4) + 18 = -6 < 0 \Rightarrow \forall x_0 = -4 \text{ lok. maximum}$$

(75)

(8.1)

$$f: y = -(1-x)(x-3)^2 = (x-1)(x-3)^2$$

$$y' = (x-3)^2 + (x-1) \cdot 2(x-3) = (x-3)(x-3+2x-2) = (x-3)(3x-5) = 0 \Leftrightarrow \underline{x_0=3 \wedge x_0=\frac{5}{3}}$$

$$y'' = (3x-5) + (x-3) \cdot 3 = 3x-5 + 3x-9 = 6x-14$$

$$y''(3) = 6 \cdot 3 - 14 = 4 > 0 \Rightarrow \nu x_0=3 \text{ lok. minimum}$$

$$\underline{y''\left(\frac{5}{3}\right) = 6 \cdot \frac{5}{3} - 14 = -4 < 0 \Rightarrow \nu x_0=\frac{5}{3} \text{ lok. maximum}}$$

g)  $f: y = -(x+1)^2(x-3)^2$

$$y' = -2(x+1)(x-3)^2 - (x+1)^2 \cdot 2(x-3) = (x+1)(x-3) \cdot [-2(x-3) - 2(x+1)] = (x+1)(x-3)(-2x+6-2x-2) =$$

$$= (x+1)(x-3)(-4x+4) = 4 \cdot (x+1)(x-3)(1-x) = 0 \Leftrightarrow \underline{x_0=-1 \wedge x_0=3 \wedge x_0=1}$$

~~(\*)~~

$$y'' = [4(x+1)(x-3)(1-x)]' = [4(x-3) \cdot (1+x)(1-x)]' = [4(x-3) \cdot (1-x^2)]' = 4(1-x^2) + 4(x-3) \cdot (-2x) = 4 - 4x^2 - 8 \cdot (x^2 - 3x) =$$

$$= 4 - 4x^2 - 8x^2 + 24x = -12x^2 + 24x + 4 = -4 \cdot (3x^2 - 6x - 1)$$

$$y''(-1) = -4 \cdot (3 \cdot 1 + 6 \cdot 1 - 1) = -4 \cdot 8 = -32 \Rightarrow \nu x_0=-1 \text{ je lok. maximum}$$

$$y''(3) = -4 \cdot (3 \cdot 9 - 6 \cdot 3 - 1) = -4 \cdot 8 = -32 \Rightarrow \nu x_0=3 \text{ je lok. maximum}$$

$$y''(1) = -4 \cdot (3 \cdot 1 - 6 \cdot 1 - 1) = -4 \cdot (-4) = 16 \Rightarrow \underline{\nu x_0=1 \text{ je lok. minimum}}$$

8.1

$$h) f: y = (2x+1)^2 \cdot (2x-1)^2$$

$$\begin{aligned} y' &= 2(2x+1) \cdot 2 \cdot (2x-1)^2 + (2x+1)^2 \cdot 2 \cdot (2x-1) \cdot 2 = (2x+1)(2x-1)[4(2x-1) + 4(2x+1)] = (2x+1)(2x-1)(8x-4+8x+4) = \\ &= (2x+1)(2x-1) \cdot 16x = 0 \Leftrightarrow \underbrace{x_0 = -\frac{1}{2}}_{}, \quad \underbrace{x_0 = \frac{1}{2}}_{}, \quad \underbrace{x_0 = 0}_{} \end{aligned}$$

$$y'' = [16(2x+1)(2x-1)x] = [16x \cdot (4x^2-1)] = 16(4x^2-1) + 16x \cdot (8x) = 16(4x^2-1+8x^2) = 16(12x^2-1)$$

$$y''(-\frac{1}{2}) = 16 \cdot (12 \cdot \frac{1}{4} - 1) = 32 > 0 \Rightarrow \forall x_0 = -\frac{1}{2} \text{ lok. minimum}$$

$$y''(\frac{1}{2}) = 16 \cdot (12 \cdot \frac{1}{4} - 1) = 32 > 0 \Rightarrow \forall x_0 = \frac{1}{2} \text{ lok. minimum}$$

$$y''(0) = 16 \cdot (12 \cdot 0 - 1) = -16 < 0 \Rightarrow \cancel{\forall x_0 = 0 \text{ lok. maximum}}$$

$$ii) f: y = \frac{1}{4}x^4 - \frac{1}{3}x^3$$

$$y' = \frac{1}{4} \cdot 4x^3 - \frac{1}{3} \cdot 3x^2 = x^3 - x^2 = x^2(x-1) = 0 \Leftrightarrow \underbrace{x_0 = 0 \wedge x_0 = 1}_{}$$

$$y'' = 3x^2 - 2x$$

$$y''(0) = 3 \cdot 0 - 2 \cdot 0 = 0$$

$$y''(1) = 3 \cdot 1 - 2 \cdot 1 = 1 > 0 \Rightarrow \cancel{\forall x_0 = 1 \text{ lok. minimum}}$$

8.1

$$f: y = x^4 + 2x^3 - 3$$

$$y' = 4x^3 + 6x^2 = 2x^2(2x+3) = 0 \Leftrightarrow x_0 = 0 \wedge x_0 = -\frac{3}{2}$$

$$y'' = 4x(2x+3) + 2x^2 \cdot 2 = 8x^2 + 12x + 4x^2 = 12x^2 + 12x = 12x(x+1)$$

$$y''(0) = 0$$

$$y''(-\frac{3}{2}) = 12 \cdot (-\frac{3}{2}) \cdot (-\frac{3}{2} + 1) = -18 \cdot (-\frac{1}{2}) = 9 > 0 \Rightarrow \underline{\underline{x_0 = -\frac{3}{2} \text{ je lok. minimum}}}$$

$$h) f: y = 1 - \frac{1}{5}x^5 - \frac{1}{4}x^4$$

$$y' = -x^4 - x^3 = -x^3 \cdot (x+1) = 0 \Leftrightarrow \underline{\underline{x_0 = 0 \wedge x_0 = -1}}$$

$$y'' = -3x^2 \cdot (x+1) - x^3 = -3x^3 - 3x^2 - x^3 = -4x^3 - 3x^2 = -x^2(4x+3)$$

$$y''(0) = 0$$

$$y''(-1) = -1 \cdot (-4+3) = 1 > 0 \Rightarrow \underline{\underline{x_0 = -1 \text{ je lok. minimum}}}$$

$$l) f: y = x^5 + x^3 + 1$$

$$y' = 5x^4 + 3x^2 = x^2(5x^2 + 3) = 5x^2 \cdot (x^2 + \frac{3}{5}) = 0 \Leftrightarrow \underline{\underline{x_0 = 0}} \vee \underline{\underline{x^2 = -\frac{3}{5}}} \neq$$

$$y'' = 10x(x^2 + \frac{3}{5}) + 5x^2 \cdot 2x = 10x^3 + 6x + 10x^3 = 20x^3 + 6x$$

$$y''(0) = 0 \Rightarrow \underline{\underline{\text{nemá lokálne extrémy}}}$$

8.2 a)  $f: y = \frac{x}{2} + \frac{2}{x} = \frac{1}{2}x + 2x^{-1}$

$$y' = \frac{1}{2} - 2x^{-2} = \frac{1}{2} - \frac{2}{x^2} = \frac{x^2 - 4}{2x^2} = 0 \Leftrightarrow x^2 = 4 \Leftrightarrow \underline{\underline{x_0 = 2 \wedge x_0 = -2}}$$

$$y'' = \frac{2x(2x^2) - (x^2 - 4) \cdot 4x}{(2x^2)^2} = \frac{4x^3 - 4x^3 + 16x}{4x^4} = \frac{16x}{4x^4} = \frac{4}{x^3}$$

$$y''(2) = \frac{4}{2^3} = \frac{1}{2} > 0 \Rightarrow \underline{\underline{v \ x_0 = 2 \text{ je lok. minimum}}}$$

$$y''(-2) = \frac{4}{(-2)^3} = -\frac{1}{2} < 0 \Rightarrow \underline{\underline{v \ x_0 = -2 \text{ je lok. maximum}}}$$

b)  $f: y = \frac{x^2}{x+3}$

$$y' = \frac{2x \cdot (x+3) - x^2 \cdot 1}{(x+3)^2} = \frac{2x^2 + 6x - x^2}{(x+3)^2} = \frac{x^2 + 6x}{(x+3)^2} = 0 \Leftrightarrow x(x+6) = 0 \Leftrightarrow \underline{\underline{x_0 = 0 \wedge x_0 = -6}}$$

$$y'' = \frac{(2x+6)(x+3)^2 - (x^2 + 6x) \cdot 2(x+3)}{(x+3)^4} = \frac{(x+3)[2(x+3)^2 - 2x^2 - 12x]}{(x+3)^4} = \frac{2x^2 + 12x + 18 - 2x^2 - 12x}{(x+3)^3} = \frac{18}{(x+3)^3}$$

$$y''(0) = \frac{18}{3^3} = \frac{2}{3} > 0 \Rightarrow \underline{\underline{v \ x_0 = 0 \text{ je lok. minimum}}}$$

$$y''(-6) = \frac{18}{(-3)^3} = -\frac{2}{3} < 0 \Rightarrow \underline{\underline{v \ x_0 = -6 \text{ je lok. maximum}}}$$

(80)

c)  $f: y = \frac{2x+1}{x^2}$

$$y' = \frac{2x^2 - (2x+1) \cdot 2x}{x^4} = \frac{2x^2 - 4x^2 - 2x}{x^4} = \frac{-2x^2 - 2x}{x^4} = \frac{-2x(x+1)}{x^3} = 0 \Leftrightarrow -2x(x+1) = 0 \Leftrightarrow x_1 = 0, x_2 = -1$$

$$y'' = \frac{-2(x^3) - (-2x-2) \cdot 3x^2}{x^6} = \frac{-2x^3 + 3x^2(2x+2)}{x^6} = \frac{-2x^3 + 6x^3 + 6x^2}{x^6} = \frac{x^2(4x+6)}{x^6} = \frac{4x+6}{x^4}$$

$$y''(-1) = \frac{4 \cdot (-1) + 6}{(-1)^4} = 2 > 0 \Rightarrow \text{lok. minimum } x_0 = -1$$

d)  $f: y = \frac{x}{1+x^2}$

$$y' = \frac{1 \cdot (1+x^2) - x \cdot 2x}{(1+x^2)^2} = \frac{1+x^2 - 2x^2}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2} = 0 \Leftrightarrow (1-x)(1+x) = 0 \Leftrightarrow x_1 = 1 \wedge x_2 = -1$$

$$y'' = \frac{-2x \cdot (1+x^2)^2 - (1-x^2) \cdot 2(1+x^2) \cdot 2x}{(1+x^2)^4} = \frac{(1+x^2) \cdot [-2x(1+x^2) - (1-x^2) \cdot 4x]}{(1+x^2)^4} = \frac{-2x - 2x^3 - 4x + 4x^3}{(1+x^2)^3} =$$

$$= \frac{2x^3 - 6x}{(1+x^2)^3}$$

$$y''(1) = \frac{2 \cdot 1 - 6 \cdot 1}{(1+1)^3} = \frac{-4}{8} = -\frac{1}{2} < 0 \Rightarrow \text{lok. maximum } x_0 = 1$$

$$y''(-1) = \frac{2 \cdot (-1) - 6 \cdot (-1)}{(1+1)^3} = \frac{4}{8} = \frac{1}{2} > 0 \Rightarrow \text{lok. minimum } x_0 = -1$$

$$8.2) \text{ e)} f: y = \frac{1+x^2}{1-x^2}$$

$$y' = \frac{2x(1-x^2) - (1+x^2) \cdot (-2x)}{(1-x^2)^2} = \frac{2x - 2x^3 + 2x + 2x^3}{(1-x^2)^2} = \frac{4x}{(1-x^2)^2} = 0 \Leftrightarrow x_0 = 0$$

$$y'' = \frac{4(1-x^2)^2 - 4x \cdot 2(1-x^2) \cdot (-2x)}{(1-x^2)^4} = \frac{(1-x^2) \cdot [4 - 4x^2 + 16x^2]}{(1-x^2)^4} = \frac{12x^2 + 4}{(1-x^2)^3}$$

$$y''(0) = \frac{4}{1} = 4 > 0 \Rightarrow \cancel{x_0=0} \text{ je lok. minimum}$$

$$f: y = 1 + \frac{1}{x^2 - x} = \frac{x^2 - x + 1}{x^2 - x}$$

$$y' = \frac{(2x-1)(x^2-x) - (x^2-x+1)(2x-1)}{(x^2-x)^2} = \frac{(2x-1)(x^2-x-x^2+x-1)}{(x^2-x)^2} = \frac{1-2x}{(x^2-x)^2} = 0 \Leftrightarrow 1-2x=0 \Leftrightarrow x_0 = \frac{1}{2}$$

$$y'' = \frac{-2(x^2-x)^2 - (1-2x) \cdot 2(x^2-x) \cdot (2x-1)}{(x^2-x)^4} = \frac{(x^2-x) \cdot [-2(x^2-x) - 2(1-2x)(2x-1)]}{(x^2-x)^4} = \frac{-2x^2 + 2x - 2(2x-1 - 4x^2 + 2x)}{(x^2-x)^3} =$$

$$= \frac{-2x^2 + 2x - 4x + 2 + 8x^2 - 4x}{(x^2-x)^3} = \frac{6x^2 - 6x + 2}{(x^2-x)^3}$$

$$y''\left(\frac{1}{2}\right) = \frac{\frac{6 \cdot \frac{1}{4} - 6 \cdot \frac{1}{2} + 2}{\left(\frac{1}{4} - \frac{1}{2}\right)^3}}{\left(-\frac{1}{4}\right)^3} = \frac{\frac{3}{2} - 3 + 2}{\left(-\frac{1}{4}\right)^3} = -\frac{64}{\frac{1}{64}} = -32 < 0 \Rightarrow \cancel{x_0=\frac{1}{2}} \text{ je lok. maximum}$$

g)  $f: y = \frac{x^3}{x^2 + 1}$

$$y' = \frac{3x^2(x^2+1) - x^3(2x)}{(x^2+1)^2} = \frac{3x^4 + 3x^2 - 2x^4}{(x^2+1)^2} = \frac{x^4 + 3x^2}{(x^2+1)^2} = \frac{x^2(x^2+3)}{(x^2+1)^2} = 0 \Leftrightarrow \boxed{x=0} \vee x^2 = -3$$

∅

$$y'' = \left( \frac{x^4 + 3x^2}{(x^2+1)^2} \right)' = \frac{(4x^3 + 6x)(x^2+1)^2 - (x^4 + 3x^2) \cdot 2 \cdot (x^2+1) \cdot 2x}{[(x^2+1)^2]^2} = \frac{(x^2+1) \cdot [(4x^3 + 6x)(x^2+1) - 4x(x^4 + 3x^2)]}{(x^2+1)^4} =$$

$$= \frac{4x^5 + 4x^3 + 6x^3 + 6x - 4x^5 - 12x^3}{(x^2+1)^3} = \frac{-2x^3 + 6x}{(x^2+1)^3}$$

$$y''(0) = \frac{0}{1} = 0 \Rightarrow \underline{\text{funkcia nemá lok. extrémy}}$$

h)  $f: y = \frac{x^4+1}{x^2}$

$$y' = \frac{4x^3 \cdot x^2 - (x^4+1) \cdot 2x}{x^4} = \frac{4x^5 - 2x^5 - 2x}{x^4} = \frac{2x(x^4-1)}{x^4} = \frac{2(x^4-1)}{x^3} = 0 \Leftrightarrow x^4 = 1 \Leftrightarrow \boxed{x_0 = -1 \wedge x_0 = 1}$$

$$y'' = \frac{2 \cdot 4x^3 \cdot x^3 - 2(x^4-1) \cdot 3x^2}{x^6} = \frac{8x^6 - 6x^2(x^4-1)}{x^6} = \frac{8x^6 - 6x^6 + 6x^2}{x^6} = \frac{2x^6 + 6x^2}{x^6} = \frac{2x^4 + 6}{x^4}$$

$$y''(-1) = \frac{2 \cdot 1 + 6}{1} = y''(1) = 8 > 0 \Rightarrow \underline{\underline{x_0 = -1 \text{ a } x_0 = 1 \text{ sú funkcia lok. minimum}}$$

(8.2)

$$ii) f: y = \frac{1}{x^4 - 1}$$

$$y' = \frac{-4x^3}{(x^4 - 1)^2} = 0 \Leftrightarrow x_0 = 0$$

$$y'' = \frac{-12x^2(x^4 - 1)^2 + 4x^3 \cdot 2(x^4 - 1) \cdot 4x^3}{(x^4 - 1)^4} = \frac{(x^4 - 1)x^2[-12(x^4 - 1) + 32x^4]}{(x^4 - 1)^4} = \frac{x^2(32x^4 - 12x^4 + 12)}{(x^4 - 1)^3} = \frac{x^2(20x^4 + 12)}{(x^4 - 1)^3}$$

$$y''(0) = \frac{0 \cdot (0+12)}{(0-1)^3} = 0 \Rightarrow \text{funkcia má mimo lok. extrémum}$$

$$j) f: y = \frac{(x+1)^2}{x^2 - 2x}$$

$$y' = \frac{2(x+1)(x^2 - 2x) - (x+1)^2 \cdot (2x-2)}{(x^2 - 2x)^2} = \frac{(x+1) \cdot (2x^2 - 4x - (2x^2 + 2))}{(x^2 - 2x)^2} = \frac{(x+1)(-4x + 2)}{(x^2 - 2x)^2} = 0 \Leftrightarrow x_0 = -1 \wedge x_0 = \frac{1}{2}$$

$$y'' = \left( \frac{(x+1)(2-4x)}{(x^2 - 2x)^2} \right)' = \left( \frac{2x - 4x^2 + 2 - 4x}{(x^2 - 2x)^2} \right)' = \left( \frac{-4x^2 - 2x + 2}{(x^2 - 2x)^2} \right)' = \frac{(-8x-2)(x^2 - 2x)^2 - (-4x^2 - 2x + 2) \cdot 2(x^2 - 2x) \cdot (2x-2)}{(x^2 - 2x)^4}$$

$$= \frac{(x^2 - 2x)[(-8x-2)(x^2 - 2x) - 2(2x-2)(-4x^2 - 2x + 2)]}{(x^2 - 2x)^4} = \frac{-8x^3 + 16x^2 - 2x^2 + 4x - 2(-8x^3 - 4x^2 + 4x + 8x^2 + 4x - 4)}{(x^2 - 2x)^3} =$$

$$= \frac{-8x^3 + 14x^2 + 4x + 16x^3 + 8x^2 - 8x - 16x^2 - 8x + 8}{(x^2 - 2x)^3} = \frac{8x^3 + 6x^2 - 12x + 8}{(x^2 - 2x)^3}$$

(84)

$$(8.2) \quad y''' = \frac{8x^3 + 6x^2 - 12x + 8}{(x^2 - 2x)^3}$$

$$y'''(-1) = \frac{-8+6+12+8}{(1+2)^3} = \frac{18}{27} = \frac{2}{3} > 0 \Rightarrow \text{v } x_0 = -1 \text{ je lok. minimum}$$

$$y'''\left(\frac{1}{2}\right) = \frac{8 \cdot \frac{1}{8} + 6 \cdot \frac{1}{4} - 12 \cdot \frac{1}{2} + 8}{\left(\frac{1}{4} - 2 \cdot \frac{1}{2}\right)^3} = \frac{1 + \frac{3}{2} - 6 + 8}{\left(\frac{1}{4} - 1\right)^3} = \frac{\frac{9}{2}}{\left(-\frac{3}{4}\right)^3} = \frac{\frac{9}{2}}{-\frac{27}{64}} = -\frac{64 \cdot 9}{2 \cdot 27} < 0 \Rightarrow \text{v } x_0 = \frac{1}{2} \text{ je lok. maximum}$$

(8.3)

$$\text{a) } f: y = x - \sqrt{x-1} = x - (x-1)^{\frac{1}{2}}$$

$$y' = 1 - \frac{1}{2}(x-1)^{-\frac{1}{2}} = 1 - \frac{1}{2\sqrt{x-1}} = \frac{2\sqrt{x-1} - 1}{2\sqrt{x-1}} = 0 \Leftrightarrow 2\sqrt{x-1} = 1$$

$$\sqrt{x-1} = \frac{1}{2} \mid^2$$

$$x-1 = \frac{1}{4}$$

$$\boxed{x_0 = \frac{5}{4}}$$

$$y'' = \left[ 1 - \frac{1}{2}(x-1)^{-\frac{1}{2}} \right]' = \frac{1}{4}(x-1)^{-\frac{3}{2}} = -\frac{1}{4\sqrt{(x-1)^3}}$$

$$y''\left(\frac{5}{4}\right) = \frac{1}{4\sqrt{\left(\frac{5}{4}-1\right)^3}} = 2 > 0 \Rightarrow \text{v } x_0 = \frac{5}{4} \text{ je lok. minimum}$$

8.3 b)  $f: y = 2x + \sqrt{2x-1} = 2x + (2x-1)^{\frac{1}{2}}$

$$y' = 2 + \frac{1}{2}(2x-1)^{-\frac{1}{2}} \cdot 2 = 2 + \frac{1}{\sqrt{2x-1}} = \frac{2\sqrt{2x-1} + 1}{\sqrt{2x-1}} = 0 \Leftrightarrow \sqrt{2x-1} = -\frac{1}{2} \Rightarrow \text{funkcia nemá lok. extrémum}$$

c)  $f: y = 4 - \sqrt[3]{x} = 4 - x^{\frac{1}{3}}$

$$y' = -\frac{1}{3}x^{-\frac{2}{3}} = -\frac{1}{3\sqrt[3]{x^2}} \neq 0 \text{ nukdy} \Rightarrow \text{funkcia nemá lok. extrémum}$$

d)  $f: y = 1 - \sqrt[3]{x^2} = 1 - x^{\frac{2}{3}}$

$$y' = -\frac{2}{3}x^{-\frac{1}{3}} = -\frac{2}{3\sqrt[3]{x}} \neq 0 \text{ nukdy} \Rightarrow \text{funkcia nemá lok. extrémum}$$

e)  $f: y = 2 - \sqrt[3]{(2-x)^2} = 2 - (2-x)^{\frac{2}{3}}$

$$y' = -\frac{2}{3}(2-x)^{-\frac{1}{3}} \cdot (-1) = \frac{2}{3\sqrt[3]{2-x}} \neq 0 \text{ nukdy} \Rightarrow \text{funkcia nemá lok. extrémum}$$

f)  $f: y = 3\sqrt[3]{(x+1)^2} - 2x = 3 \cdot (x+1)^{\frac{2}{3}} - 2x$

$$y' = 3 \cdot \frac{2}{3} \cdot (x+1)^{-\frac{1}{3}} - 2 = \frac{2}{\sqrt[3]{x+1}} - 2 = \frac{2 - 2\sqrt[3]{x+1}}{\sqrt[3]{x+1}} = 0 \Leftrightarrow \sqrt[3]{x+1} = 1 / 1^3$$

$$x+1=1$$

$$\boxed{x_0 = 0}$$

$$(8_3) f) y''' = \left( 2(x+1)^{-\frac{1}{3}} - 2 \right) = 2 \cdot \left( -\frac{1}{3} \right) (x+1)^{-\frac{4}{3}} = \frac{-2}{3 \cdot \sqrt[3]{(x+1)^4}}$$

$$y'''(0) = \frac{-2}{3 \cdot \sqrt[3]{1}} = -\frac{2}{3} < 0 \Rightarrow \text{v } x_0 \text{ má funkcia lok. maximum}$$

(86)

$$g) f: y = x \cdot \sqrt{9-x} = x(9-x)^{\frac{1}{2}}$$

$$y' = (9-x)^{\frac{1}{2}} + x \cdot \frac{1}{2}(9-x)^{-\frac{1}{2}}(-1) = \sqrt{9-x} - \frac{x}{2} \cdot \frac{1}{\sqrt{9-x}} = \frac{2(9-x)-x}{2\sqrt{9-x}} = \frac{18-2x-x}{2\sqrt{9-x}} = \frac{18-3x}{2\sqrt{9-x}} = 0 \Leftrightarrow x_0 = 6$$

$$y''' = \left[ (9-x)^{\frac{1}{2}} - \frac{x}{2} \cdot (9-x)^{-\frac{1}{2}} \right] = -\frac{1}{2}(9-x)^{-\frac{1}{2}} - \left( \frac{1}{2}(9-x)^{\frac{1}{2}} + \frac{x}{2} \cdot \left( -\frac{1}{2} \right)(9-x)^{-\frac{3}{2}} \cdot (-1) \right) = \frac{-1}{2\sqrt{9-x}} - \frac{1}{2\sqrt{9-x}} - \frac{x}{4\sqrt{(9-x)^3}} =$$

$$= \frac{-2(9-x) - 2(9-x) - x}{4\sqrt{(9-x)^3}} = \frac{-36 + 3x}{4\sqrt{(9-x)^3}}$$

$$y'''(6) = \frac{-36 + 18}{4\sqrt{27}} = \frac{-18}{4\sqrt{27}} < 0 \Rightarrow \text{v } x_0 = 6 \text{ má funkcia lok. maximum}$$

$$h) f: y = (5-x) \cdot \sqrt[3]{x^2} = (5-x) \cdot x^{\frac{2}{3}}$$

$$y' = (-1) \cdot x^{\frac{2}{3}} + (5-x) \cdot \frac{2}{3}x^{-\frac{1}{3}} = \cancel{\left( \frac{-x}{\sqrt[3]{x^2}} + \frac{2(5-x)}{3\sqrt[3]{x^2}} \right)} - \cancel{\frac{-3 + (10-2x)\sqrt[3]{x^2}}{3\sqrt[3]{x^2}}} \\ - \sqrt[3]{x^2} + \frac{2(5-x)}{3\sqrt[3]{x^2}} = \frac{-3\sqrt[3]{x} \cdot \sqrt[3]{x^2} + 10-2x}{3\sqrt[3]{x^3}} = \frac{-3x-2x+10}{3\sqrt[3]{x^3}} = \frac{-5x+10}{3\sqrt[3]{x^3}} = 0 \\ \Leftrightarrow x_0 = 2$$

(803)

$$h) \quad y''' = \left( \frac{-5x+10}{3\sqrt[3]{x^4}} \right)' = \left[ \frac{1}{3} x^{-\frac{1}{3}} \cdot (10-5x) \right]' = \left( \frac{1}{3} x^{-\frac{1}{3}} (10-5x) \right)' = \frac{1}{3} \cdot \left( -\frac{1}{3} \right) x^{-\frac{4}{3}} \cdot (10-5x) + \frac{1}{3} x^{-\frac{1}{3}} \cdot (-5) =$$

$$= \frac{10-5x}{-9 \cdot \sqrt[3]{x^4}} - \frac{5}{3\sqrt[3]{x^4}} = \frac{5x-10}{9 \cdot \sqrt[3]{x^4}} - \frac{5}{3\sqrt[3]{x^4}} = \frac{5x-10-5 \cdot (3x)}{9 \cdot \sqrt[3]{x^4}} = \frac{-10x-10}{9 \cdot \sqrt[3]{x^4}}$$

$$y'''(2) = \frac{-10 \cdot 2 - 10}{9 \cdot \sqrt[3]{16}} = \frac{-30}{9 \cdot \sqrt[3]{16}} < 0 \Rightarrow \underline{\underline{v \ x_0=2 \text{ je lok. maximum}}}$$

(87)

8.4

a)  $f: y = x \cdot e^{-x}$

$$y' = e^{-x} + x \cdot e^{-x} \cdot (-1) = e^{-x}(1-x) = 0 \Leftrightarrow \underline{x_0 = 1}$$

$$y'' = e^{-x}(-1)(1-x) + e^{-x} \cdot (-1) = -e^{-x}(1-x+1) = -e^{-x}(2-x) = e^{-x}(x-2)$$

$$y''(1) = e^{-1} \cdot (-1) = -\frac{1}{e} < 0 \Rightarrow \underline{v \ x_0 = 1 \text{ je lok. maximum}}$$

b)  $f: y = (4-x) e^{4-x}$

$$y' = (-1) \cdot e^{4-x} + (4-x) e^{4-x} \cdot (-1) = e^{4-x}(-1-4+x) = e^{4-x}(x-5) = 0 \Leftrightarrow \underline{x_0 = 5}$$

$$y'' = e^{4-x} \cdot (-1)(x-5) + e^{4-x} \cdot 1 = e^{4-x}(-x+5+1) = e^{4-x} \cdot (6-x)$$

$$y''(5) = e^{-1} \cdot 1 = \frac{1}{e} > 0 \Rightarrow \underline{v \ x_0 = 5 \text{ je lok. minimum}}$$

c)  $f: y = (x^2+1) \cdot e^{-x}$

$$y' = 2x e^{-x} + (x^2+1) e^{-x} \cdot (-1) = e^{-x}(2x-x^2-1) = e^{-x} \cdot (-x^2+2x-1) = 0 \Leftrightarrow -x^2+2x-1=0 \ | \cdot (-1)$$

$$x^2-2x+1=0$$

$$(x-1)^2=0 \Leftrightarrow \underline{x_0 = 1}$$

$$y'' = e^{-x} \cdot (-1) \cdot (-x^2+2x-1) + e^{-x} \cdot (-2x+2) = e^{-x} \cdot (x^2-2x+1-2x+2) = e^{-x} \cdot (x^2-4x+3)$$

$$y''(1) = e^{-1} \cdot (1-4+3) = 0 \Rightarrow \underline{\text{lok. extremum nichtig}}$$

8.4) d)  $f: y = \bar{e}^x$

$$y' = \bar{e}^x \cdot (-2x) = -2x\bar{e}^{-x} = 0 \Leftrightarrow \underline{x_0 = 0}$$

$$y'' = -2\bar{e}^{-x} + (-2x) \cdot \bar{e}^{-x} \cdot (-2) = \bar{e}^{-x} \cdot (-2 + 4x^2) = \bar{e}^{-x} \cdot (4x^2 - 2)$$

$$y''(0) = \bar{e}^0 \cdot (-2) = -2 < 0 \Rightarrow \underline{v x_0 = 0 \text{ je lok. maximum}}$$

e)  $f: y = x \cdot e^{\frac{x^2}{2}}$

$$y' = e^{\frac{x^2}{2}} + x \cdot e^{\frac{x^2}{2}} \cdot \left(\frac{1}{2} \cdot 2x\right) = e^{\frac{x^2}{2}} \cdot (1+x^2) \neq 0 \text{ nikdy} \Rightarrow \underline{\text{funkcia nemá lok. extrém}}$$

f)  $f: y = x + \bar{e}^{-x}$

$$y' = 1 - \bar{e}^{-x} = \frac{\bar{e}^x - 1}{\bar{e}^x} = 0 \Leftrightarrow \bar{e}^x = 1 \Leftrightarrow \underline{x_0 = 0}$$

$$y'' = (1 - \bar{e}^{-x})' = \bar{e}^{-x}$$

$$y''(0) = \bar{e}^0 = 1 > 0 \Rightarrow \underline{v x_0 = 0 \text{ je lok. minimum}}$$

g)  $f: y = e^x + \bar{e}^{-x}$

$$y' = e^x - \bar{e}^{-x} = 0 \Leftrightarrow e^x = \bar{e}^{-x} \\ x = -x \Rightarrow \underline{x_0 = 0}$$

$$y'' = e^x + \bar{e}^{-x}$$

$$y''(0) = e^0 + \bar{e}^0 = 2 > 0 \Rightarrow \underline{v x_0 = 0 \text{ je lok. minimum}}$$

(8.4) h)  $f: y = \frac{e^x}{x+1}$

$$y' = \frac{e^x(x+1) - e^x \cdot 1}{(x+1)^2} = \frac{e^x \cdot x}{(x+1)^2} = 0 \Leftrightarrow x_0 = 0$$

$$y'' = \frac{(e^x \cdot x + e^x \cdot 1)(x+1)^2 - e^x \cdot x \cdot 2(x+1)}{(x+1)^4} = \frac{e^x(x+1)[(x+1)^2 - 2x]}{(x+1)^4} = \frac{e^x(x+1)(x^2+2x+1-2x)}{(x+1)^4} = \frac{e^x(x+1)(x^2+1)}{(x+1)^3}$$

$$y''(0) = \frac{e^0 \cdot (0+1)}{(0+1)^3} = \frac{1 \cdot 1}{1} = 1 > 0 \Rightarrow \underline{\underline{x_0=0 \text{ je lok. minimum}}}$$

(9.1)

(8.5) a)  $f: y = x \ln x$

$$y' = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1 = 0 \Leftrightarrow \ln x = -1$$

$$e^{\ln x} = e^{-1}$$

$$x = e^{-1} \Rightarrow \underline{\underline{x_0 = \frac{1}{e}}}$$

$$y'' = \frac{1}{x}$$

$$y''\left(\frac{1}{e}\right) = \frac{1}{\frac{1}{e}} = e > 0 \Rightarrow \underline{\underline{x_0 = \frac{1}{e} \text{ je lok. minimum}}}$$

b)  $f: y = \frac{x^2}{2} - \ln x$

$$y' = \frac{1}{2} \cdot 2x - \frac{1}{x} = x - \frac{1}{x} = \frac{x^2 - 1}{x} = \frac{(x+1)(x-1)}{x} = 0 \Leftrightarrow \underline{\underline{x_0 = -1 \wedge x_0 = 1}}$$

$$y'' = \left(\frac{x^2 - 1}{x}\right)' = \frac{2x \cdot x - (x^2 - 1) \cdot 1}{x^2} = \frac{2x^2 - x^2 + 1}{x^2} = \frac{x^2 + 1}{x^2}$$

$$y''(-1) = \frac{1+1}{1} = 2 > 0 \quad \left. \begin{array}{l} \\ \end{array} \right\} \Rightarrow \underline{\underline{x_0 = -1 \text{ w. } x_0 = 1 \text{ je lok. minimum}}}$$

$$y''(1) = \underline{\underline{\quad / / \quad}}$$

9.1

c)  $f: y = \frac{1}{x} + \ln x = x^{-1} + \ln x$

$$y' = -x^{-2} + \frac{1}{x} = \frac{-1+x}{x^2} = 0 \Leftrightarrow \underline{x_0=1}$$

$$y'' = \frac{1 \cdot x^2 - (-1+x) \cdot 2x}{x^4} = \frac{x^2 + 2x - 2x^2}{x^4} = \frac{-x^2 + 2x}{x^4} = \frac{x(2-x)}{x^4} = \frac{2-x}{x^3}$$

$$y''(1) = \frac{2-1}{1} = 1 > 0 \Rightarrow \underline{\underline{x_0=1 \text{ je lok. minimum}}}$$

d)  $f: y = \frac{1+\ln x}{x}$

$$y' = \frac{\frac{1}{x} \cdot x - (1+\ln x) \cdot 1}{x^2} = \frac{1-1-\ln x}{x^2} = \frac{-\ln x}{x^2} = 0 \Leftrightarrow -\ln x = 0 \mid \cdot(-1)$$

$$\begin{aligned}\ln x &= 0 \\ e^{\ln x} &= e^0 \Leftrightarrow \underline{x_0=1}\end{aligned}$$

$$y'' = \frac{\left(-\frac{1}{x}\right)x^2 - (-\ln x) \cdot 2x}{x^4} = \frac{-x + 2x\ln x}{x^4} = \frac{x(2\ln x - 1)}{x^4} = \frac{2\ln x - 1}{x^3}$$

$$y''(1) = \frac{2 \cdot \ln 1 - 1}{1} = \frac{0-1}{1} = -1 < 0 \Rightarrow \underline{\underline{x_0=1 \text{ je lok. maximum}}}$$

e)  $f: y = \ln(4x-x^2)$

$$y' = \frac{1}{4x-x^2} \cdot (4-2x) = \frac{4-2x}{4x-x^2} = 0 \Leftrightarrow 4-2x=0 \Leftrightarrow \underline{x_0=2}$$

$$y'' = \frac{-2(4x-x^2) - (4-2x)(4-2x)}{(4x-x^2)^2} = \frac{-8x+2x^2-16+16x-4x^2}{(4x-x^2)^2} = \frac{-2x^2+8x-16}{(4x-x^2)^2}$$

$$y''(2) = \frac{-2 \cdot 4 + 8 \cdot 2 - 16}{(8-4)^2} = \frac{-8}{16} = -\frac{1}{2} < 0 \Rightarrow \underline{\underline{x_0=2 \text{ je lok. maximum}}}$$

9.1) f)  $y = \ln x - 2 \ln x$

$$y' = 2 \ln x \cdot \frac{1}{x} - 2 \cdot \frac{1}{x} = \frac{2 \ln x - 2}{x} = 0 \Leftrightarrow 2 \ln x = 2$$

$$\ln x = 1 \Leftrightarrow x_0 = e$$

$$y'' = \frac{2 \cdot \frac{1}{x} \cdot x - (2 \ln x - 2) \cdot 1}{x^2} = \frac{2 - 2 \ln x + 2}{x^2} = \frac{4 - 2 \ln x}{x^2}$$

$$y''(e) = \frac{4 - 2 \cdot \ln e}{e^2} = \frac{4 - 2}{e^2} = \frac{2}{e^2} > 0 \Rightarrow \underline{\underline{x_0 = e \text{ je lok. minimum}}}$$

g) f)  $y = \ln \left( \frac{x-1}{x+1} \right)$

$$y' = \frac{1}{\frac{x-1}{x+1}} \cdot \frac{(x+1) - (x-1)}{(x+1)^2} = \frac{x+1}{x-1} \cdot \frac{2}{(x+1)^2} = \frac{2}{(x-1)(x+1)} \neq 0 \text{ nikdy} \Rightarrow \underline{\underline{\text{funkcia nemá lok. extrém}}}$$

h) f)  $y = \frac{\ln x}{x^2}$

$$y' = \frac{\frac{1}{x} \cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{x - 2x \ln x}{x^4} = \frac{x(1 - 2 \ln x)}{x^4} = \frac{1 - 2 \ln x}{x^3} = 0 \Leftrightarrow 1 = 2 \ln x$$

$$\ln x = \frac{1}{2} \Rightarrow \underline{\underline{x_0 = \sqrt{e}}}$$

$$y'' = \frac{-2 \cdot \frac{1}{x} \cdot x^3 - (1 - 2 \ln x) \cdot 3x^2}{x^6} = \frac{-2x^2 - 3x^2 + 6x^2 \ln x}{x^6} = \frac{x^2(-5 + 6 \ln x)}{x^6} = \frac{-5 + 6 \ln x}{x^4}$$

$$y''(\sqrt{e}) = \frac{-5 + 6 \cdot \ln e^{\frac{1}{2}}}{(\sqrt{e})^4} = \frac{-5 + 6 \cdot \frac{1}{2}}{e^2} = \frac{-5 + 3}{e^2} = \frac{-2}{e^2} < 0 \Rightarrow \underline{\underline{x_0 = \sqrt{e} \text{ je lok. maximum}}}$$

$$i) f: y = \frac{\ln x}{\sqrt{2x}} = \frac{\ln x}{(2x)^{\frac{1}{2}}}$$

$$\begin{aligned} y' &= \frac{\frac{1}{x} \cdot (2x)^{\frac{1}{2}} - \ln x \cdot \frac{1}{2}(2x)^{-\frac{1}{2}} \cdot 2}{2x} = \frac{\frac{1}{x} \cdot \sqrt{2x} - \frac{\ln x}{\sqrt{2x}}}{2x} = \frac{\frac{\sqrt{2x} \cdot \sqrt{2x} - x \ln x}{x \sqrt{2x}}}{2x} = \frac{2x - \ln x \cdot x}{2x^2 \sqrt{2x}} = \frac{x(2 - \ln x)}{2x^2 \sqrt{2x}} = \\ &= \frac{2 - \ln x}{2x \sqrt{2x}} = 0 \Leftrightarrow 2 = \ln x \end{aligned}$$

$$e^{\ln x} = e^2 \Leftrightarrow x_0 = e^2$$

$$\frac{a^2}{a} = a$$

$$\begin{aligned} y'' &= \frac{-\frac{1}{x} \cdot 2 \cdot \sqrt{2x} - (2 - \ln x) \cdot (2\sqrt{2x} + 2x \cdot \frac{1}{2} \cdot (2x)^{-\frac{1}{2}} \cdot 2)}{(2\sqrt{2x})^2} = \frac{-2\sqrt{2x} + (\ln x - 2) \left( 2\sqrt{2x} + \frac{2x}{\sqrt{2x}} \right)}{4x^2 \cdot 2x} = \\ &= \frac{-2\sqrt{2x} + (\ln x - 2)(2\sqrt{2x} + \sqrt{2x})}{8x^3} = \frac{-2\sqrt{2x} + (\ln x - 2) \cdot 3\sqrt{2x}}{8x^3} = \frac{\sqrt{2x} \cdot (-2 + 3\ln x - 6)}{8x^3} = \\ &= \frac{\sqrt{2x} \cdot (3\ln x - 8)}{8x^3} \end{aligned}$$

$$y''(e^2) = \frac{\sqrt{2e^2} \cdot (3\ln e^2 - 8)}{8 \cdot (e^2)^3} = \frac{\sqrt{2}e \cdot (3 \cdot 2 - 8)}{8e^6} = \frac{\sqrt{2} \cdot (-2)}{8e^5} = \frac{-\sqrt{2}}{4e^5} < 0 \Rightarrow v \ x_0 = e^2 \text{ je lok. max.}$$

$$j) f: y = \frac{\ln^2 x}{x}$$

$$y' = \frac{2\ln x \cdot \frac{1}{x} \cdot x - \ln^2 x \cdot 1}{x^2} = \frac{2\ln x - \ln^2 x}{x^2} = \frac{\ln x \cdot (2 - \ln x)}{x^2} = 0 \Leftrightarrow \ln x = 0 \vee \ln x = 2$$

$$| x_0 = 1 \vee x_0 = e^2 |$$

$$y'' = \left( \frac{2\ln x - \ln^2 x}{x^2} \right)' = \frac{\left( 2 \cdot \frac{1}{x} - 2\ln x \cdot \frac{1}{x} \right)x^2 - (2\ln x - \ln^2 x) \cdot 2x}{x^4} = \textcircled{*}$$

$$\textcircled{q.1} \quad j) \quad \textcircled{*} = \frac{\frac{2-2\ln x}{x} \cdot x^2 - 2x(2\ln x - \ln^2 x)}{x^4} = \frac{x[2-2\ln x - 4\ln x + 2\ln^2 x]}{x^4} = \frac{2\ln^2 x - 6\ln x + 2}{x^3}$$

$$y''(1) = \frac{2 \cdot \ln^2 1 - 6 \cdot \ln 1 + 2}{1} = \frac{2}{1} = 2 > 0 \Rightarrow \forall x_0 = 1 \text{ je lok. minimum}$$

$$y''(e^2) = \frac{2\ln^2 e^2 - 6\ln e^2 + 2}{(e^2)^3} = \frac{2 \cdot 4 - 6 \cdot 2 + 2}{e^6} = \frac{-2}{e^6} < 0 \Rightarrow \forall x_0 = e^2 \text{ je lok. maximum}$$

$$\textcircled{3.1.1} \quad a) \int (5x^2 - 4x + 10) dx = 5 \cdot \frac{x^3}{3} - 4 \cdot \frac{x^2}{2} + 10x = \frac{5}{3}x^3 - 2x^2 + 10x + C; c \in \mathbb{R}$$

$$b) \int x \cdot (3x-4)^2 dx = \int x \cdot (9x^2 - 24x + 16) dx = \int (9x^3 - 24x^2 + 16x) dx = 9 \cdot \frac{x^4}{4} - 24 \cdot \frac{x^3}{3} + 16 \cdot \frac{x^2}{2} = \frac{9}{4}x^4 - 8x^3 + 8x^2 + C; c \in \mathbb{R}$$

$$c) \int (x^2 - 4x \sqrt[3]{x} + 10 \sqrt[4]{x^3}) dx = \int \left( x^2 - 4x \cdot x^{\frac{1}{3}} + 10 \cdot x^{\frac{3}{4}} \right) dx = \int \left( x^2 - 4x^{\frac{7}{3}} + 10x^{\frac{3}{4}} \right) dx = \frac{x^3}{3} - 4 \cdot \frac{x^{\frac{10}{3}}}{\frac{10}{3}} + 10 \cdot \frac{x^{\frac{7}{4}}}{\frac{7}{4}} + C = \frac{x^3}{3} - 4 \cdot \frac{3}{7} \cdot x^{\frac{7}{3}} + 10 \cdot \frac{4}{7} \cdot x^{\frac{7}{4}} + C; c \in \mathbb{R}$$

$$d) \int (x-2)(4-x) dx = \int (4x - x^2 - 8 + 2x) dx = \int (-x^2 + 6x - 8) dx = -\frac{x^3}{3} + 6 \cdot \frac{x^2}{2} - 8x = -\frac{1}{3}x^3 + 3x^2 - 8x + C; c \in \mathbb{R}$$

$$e) \int (x^2-2)^3 dx = \int (x^2-2)^2 \cdot (x^2-2) dx = \int (x^4 - 4x^2 + 4)(x^2-2) dx = \int (x^6 - 2x^4 - 4x^4 + 8x^2 + 4x^2 - 8) dx = \int (x^6 - 6x^4 + 12x^2 - 8) dx = \frac{x^7}{7} - 6 \cdot \frac{x^5}{5} + 12 \cdot \frac{x^3}{3} - 8x + C = \frac{1}{7}x^7 - \frac{6}{5}x^5 + 4x^3 - 8x + C; c \in \mathbb{R}$$

3.1.1

$$f) \int (x^2 - \frac{5}{2}x + 6)(x^3 + 1) dx = \int (x^5 + x^2 - \frac{5}{2}x^4 - \frac{5}{2}x^3 + 6x^3 + 6) dx = \int (x^5 - \frac{5}{2}x^4 + 6x^3 + x^2 - \frac{5}{2}x + 6) dx =$$

$$= \frac{x^6}{6} - \frac{5}{2} \cdot \frac{x^5}{5} + 6 \cdot \frac{x^4}{4} + \frac{x^3}{3} - \frac{5}{2} \cdot \frac{x^2}{2} + 6x + C = \underline{\underline{\frac{1}{6}x^6 - \frac{1}{2}x^5 + \frac{3}{2}x^4 + \frac{1}{3}x^3 - \frac{5}{4}x^2 + 6x + C; C \in \mathbb{R}}}$$

$$\begin{aligned} g) \int \left( \frac{3}{x} - \frac{7}{x^3} + \frac{6}{\sqrt[3]{x^7}} - \frac{18}{\sqrt{x^1}} \right) dx &= \int \left( 3 \cdot \frac{1}{x} - 7 \cdot \frac{1}{x^3} + 6 \cdot \frac{1}{x^{\frac{1}{3}}} - 18 \cdot \frac{1}{x \cdot x^{\frac{1}{2}}} \right) dx = \int \left( 3x^{-1} - 7x^{-3} + 6x^{-\frac{1}{3}} - 18x^{-\frac{3}{2}} \right) dx = \\ &= 3 \ln x - 7 \cdot \frac{x^{-2}}{-2} + 6 \cdot \frac{x^{\frac{2}{3}}}{\frac{2}{3}} - 18 \cdot \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + C = \\ &= 3 \ln x + \frac{7}{2} \cdot \frac{1}{x^2} + 6 \cdot \frac{3}{2} \cdot \sqrt[3]{x^2} + 18 \cdot 2 \cdot \frac{1}{\sqrt{x}} + C = 3 \ln x + \frac{7}{2x^2} + 9\sqrt[3]{x^2} + \frac{36}{\sqrt{x}} + C \end{aligned}$$

$$h) \int (x \cdot \sqrt{x - \sqrt{x^3}}) dx = \int x \cdot (x - x^{\frac{1}{2}})^{\frac{1}{2}} dx = \int x \cdot x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} dx = \int x^{1 + \frac{1}{2} + \frac{1}{4}} dx = \int x^{\frac{4+2+1}{4}} dx = \int x^{\frac{7}{4}} dx = \frac{x^{\frac{11}{4}}}{\frac{11}{4}} = \underline{\underline{\frac{4}{11} \cdot \sqrt[4]{x^{11}}}} + C$$

$$\text{i) } \int (1-3x+x^3)(\sqrt[3]{x}) dx = \int (1-3x+x^3) \cdot x^{\frac{1}{3}} dx = \int \left( x^{\frac{1}{3}} - 3 \cdot x^{\frac{4}{3}} + x^{\frac{10}{3}} \right) dx = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} - 3 \cdot \frac{x^{\frac{7}{3}}}{\frac{7}{3}} + \frac{x^{\frac{13}{3}}}{\frac{13}{3}} + C = \frac{3}{4} \cdot \sqrt[3]{x^4} - 3 \cdot \frac{3}{7} \cdot \sqrt[3]{x^7} + \frac{3}{13} \cdot \sqrt[3]{x^{13}} + C; C \in \mathbb{R}$$

$$j) \int \sqrt{x \cdot \sqrt{x^3}} dx = \int \left(x \cdot x^{\frac{3}{2}}\right)^{\frac{1}{2}} dx = \int \left(x^{\frac{5}{2}}\right)^{\frac{1}{2}} dx = \int x^{\frac{5}{4}} dx = \frac{x^{\frac{9}{4}}}{\frac{9}{4}} + C = \frac{4}{9} \cdot \sqrt[4]{x^9} + C; C \in \mathbb{R}$$

(3.10.2)

$$a) \int (e^x - e^{3x}) dx = \underline{\underline{e^x - e^{3x} + c; c \in \mathbb{R}}}$$

$$b) \int (5 \cdot e^x) dx = \int (5e)^x dx = \underbrace{\left( \begin{array}{l} \text{Subst.:} \\ t = 5 \end{array} \right)}_{\underline{\underline{\frac{(5e)^x}{\ln(5e)} + c; c \in \mathbb{R}}}}$$

$$c) \int \frac{15^x - 9^x}{3^x} dx = \int \left( \frac{15^x}{3^x} - \frac{9^x}{3^x} \right) dx = \int \left[ \left( \frac{15}{3} \right)^x - \left( \frac{9}{3} \right)^x \right] dx = \int (5^x - 3^x) dx = \underline{\underline{\frac{5^x}{\ln 5} - \frac{3^x}{\ln 3} + c; c \in \mathbb{R}}}$$

$$d) \int \frac{(3^x + 4^x)^2}{12^x} dx = \int \frac{(3^x)^2 + 2 \cdot 3^x \cdot 4^x + (4^x)^2}{12^x} dx = \int \frac{(3^2)^x + 2 \cdot (3 \cdot 4)^x + (4^2)^x}{12^x} dx = \int \left[ \left( \frac{9}{12} \right)^x + 2 + \left( \frac{16}{12} \right)^x \right] dx = \int \left[ \left( \frac{3}{4} \right)^x + 2 + \left( \frac{4}{3} \right)^x \right] dx =$$

$$= \underline{\underline{\frac{\left( \frac{3}{4} \right)^x}{\ln \frac{3}{4}} + 2x + \frac{\left( \frac{4}{3} \right)^x}{\ln \frac{4}{3}} + c; c \in \mathbb{R}}}$$

(3.10.3)

$$a) \int (\sin x - 3 \cos x) dx = \int \sin x dx - 3 \cdot \int \cos x dx = \underline{\underline{-\cos x - 3 \cdot \sin x + c; c \in \mathbb{R}}}$$

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

$$b) \int (\cot^2 x) dx = \int \left( \frac{\cos x}{\sin x} \right)^2 dx = \int \frac{\cos^2 x}{\sin^2 x} dx \stackrel{\text{if } \frac{1-\sin^2 x}{\sin^2 x}}{=} \int \frac{1-\sin^2 x}{\sin^2 x} dx = \int \frac{1}{\sin^2 x} dx - \int 1 dx = \underline{\underline{-\cot x - x + c; c \in \mathbb{R}}}$$

$$c) \int \frac{1}{\cos^2 x \cdot \sin^2 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \cdot \sin^2 x} dx = \int \frac{\sin^2 x}{\cos^2 x \cdot \sin^2 x} dx + \int \frac{\cos^2 x}{\cos^2 x \cdot \sin^2 x} dx = \int \frac{1}{\cos^2 x} dx + \int \frac{1}{\sin^2 x} dx = \underline{\underline{\operatorname{tg} x - \operatorname{ctg} x + c}}$$

$$d) \int \frac{2}{x^2 + 9} dx = 2 \cdot \int \frac{1}{9 \cdot \left( \frac{x^2}{9} + 1 \right)} dx = \frac{2}{9} \int \frac{1}{\left( \frac{x^2}{3} + 1 \right)} dx = \left| \begin{array}{l} \text{Subst.:} \\ \frac{x^2}{3} = t \\ \frac{1}{3} dx = dt \end{array} \right| = \frac{2}{9} \cdot \int \frac{1}{t^2 + 1} \cdot 3 dt = \frac{2}{3} \arctg t = \underline{\underline{\frac{2}{3} \arctg \frac{x}{3} + c}}$$

3.1.4

(97)

$$\text{a) } \int \frac{\cos x}{10 + \sin x} dx = \begin{cases} \text{subst:} \\ 10 + \sin x = t \\ (10 + \sin x)' dx = dt \\ \cos x dx = dt \end{cases} = \int \frac{1}{t} dt = \ln|t| + c = \ln|10 + \sin x| + c; c \in \mathbb{R}$$

$$\text{b) } \int \frac{\sin x}{2 + 5 \cos x} dx = -\frac{1}{5} \int \frac{-5 \sin x}{2 + 5 \cos x} dx = -\frac{1}{5} \ln|2 + 5 \cos x| + c; c \in \mathbb{R} \rightarrow \text{lebo: } \boxed{\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c}$$

↓  
vynásobíme aj výdeľivé číslo (-5),  
aby sme v čitateľi dostali deriváciu menovateľa

$$\text{c) } \int \frac{1}{x \ln x} dx = \begin{cases} \ln x = t \\ \frac{1}{x} dx = dt \end{cases} = \int \frac{1}{t} dt = \ln|t| + c = \ln|\ln x| + c; c \in \mathbb{R}$$

$$\text{d) } \int \frac{1}{\sin x \cdot \cos x} dx = \int \frac{\sin^2 x + \cos^2 x}{\sin x \cdot \cos x} dx = \int \left( \frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x} \right) dx = \int \left( \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) dx = \int \left( \frac{1}{\cos x} + \frac{1}{\sin x} \right) dx = -\ln|\cos x| + \ln|\sin x| + c$$

3.1.5

$$a) \int \frac{3}{2-5x} dx = -\frac{3}{5} \int \frac{-5}{2-5x} dx = -\frac{3}{5} \ln |2-5x| + c; c \in \mathbb{R}$$

$$b) \int \frac{x}{9+4x^2} dx = \frac{1}{8} \int \frac{8x}{9+4x^2} dx = \frac{1}{8} \ln |9+4x^2| + c; c \in \mathbb{R}$$

3.2.1

$$a) \int \frac{\ln^4 x}{x} dx = \left| \begin{array}{l} \ln x = t \\ \frac{1}{x} dx = dt \end{array} \right| = \int t^4 dt = \frac{t^5}{5} + c = \frac{\ln^5 x}{5} + c; c \in \mathbb{R}$$

$$b) \int \frac{1}{x^2} \cdot \cos \frac{1}{x} dx = \left| \begin{array}{l} \frac{1}{x} = t \\ -\frac{1}{x^2} dx = dt \end{array} \right| = - \int \cos t dt = -\sin t + c = -\sin \frac{1}{x} + c; c \in \mathbb{R}$$

$$c) \int x^2 \cdot e^{x^3} dx = \left| \begin{array}{l} x^3 = t \\ 3x^2 dx = dt \end{array} \right| = \frac{1}{3} \int 3x^2 e^{x^3} dx = \frac{1}{3} \int e^t dt = \frac{1}{3} e^t + c = \frac{1}{3} e^{x^3} + c; c \in \mathbb{R}$$

$$d) \int x \cdot (3x^2 - 4)^5 dx = \left| \begin{array}{l} 3x^2 - 4 = t \\ 6x dx = dt \end{array} \right| = \frac{1}{6} \int 6x \cdot (3x^2 - 4)^5 dx = \frac{1}{6} \int t^5 dt = \frac{1}{6} \cdot \frac{t^6}{6} + c = \frac{(3x^2 - 4)^6}{36} + c; c \in \mathbb{R}$$

$$e) \int \frac{x}{(x^2 - 4)^3} dx = \left| \begin{array}{l} x^2 - 4 = t \\ 2x dx = dt \end{array} \right| = \frac{1}{2} \int \frac{1}{t^3} dt = \frac{1}{2} \int t^{-3} dt = \frac{1}{2} \cdot \frac{t^{-2}}{-2} + c = \frac{-1}{4 \cdot (x^2 - 4)^2} + c; c \in \mathbb{R}$$

3.2.1 f)  $\int x \cdot 3\sqrt{6-x^3} dx = \begin{cases} 6-x^3=t \\ -3x^2dx=dt \end{cases} = -\frac{1}{3} \int \sqrt[3]{t} dt = -\frac{1}{3} \int t^{\frac{1}{3}} dt = -\frac{1}{3} \cdot \frac{t^{\frac{4}{3}}}{\frac{4}{3}} + C = -\frac{1}{4} \cdot \underline{\underline{\sqrt[3]{(6-x^3)^4}}} + C$

g)  $\int \frac{1}{x^3} \cdot \sin \frac{1}{x^2} dx = \int x^{-3} \cdot \sin x^{-2} dx = \begin{cases} x^{-2}=t \\ -2x^{-3}dx=dt \end{cases} = -\frac{1}{2} \int \sin t dt = +\frac{1}{2} \underline{\underline{\cos t}} \frac{1}{x^2} + C; C \in \mathbb{R}$

h)  $\int \frac{3x^3}{\sqrt[3]{x^4+4}} dx = \begin{cases} x^{4+4}=t \\ 4x^3dx=dt \end{cases} = \frac{3}{4} \int \frac{4x^3}{(x^4+4)^{\frac{1}{3}}} dx = \frac{3}{4} \cdot \int \frac{1}{t^{\frac{1}{3}}} dt = \frac{3}{4} \cdot \int t^{-\frac{1}{3}} dt = \frac{3}{4} \cdot \frac{t^{\frac{2}{3}}}{\frac{2}{3}} = \underline{\underline{\frac{9}{8} \sqrt[3]{(x^4+4)^2}}} + C$

3.2.2 a)  $\int \sqrt{5+2x} dx = \begin{cases} 5+2x=t \\ 2dx=dt \end{cases} = \frac{1}{2} \int t^{\frac{1}{2}} dt = \frac{1}{2} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \underline{\underline{\frac{1}{3} \sqrt{(5+2x)^3}}} + C; C \in \mathbb{R}$

b)  $\int \frac{5}{\sqrt[3]{1-6x}} dx = \begin{cases} 1-6x=t \\ -6dx=dt \end{cases} = -\frac{5}{6} \int \frac{1}{t^{\frac{1}{3}}} dt = -\frac{5}{6} \cdot \frac{t^{\frac{2}{3}}}{\frac{2}{3}} + C = -\frac{5}{4} \cdot \underline{\underline{\sqrt[3]{(1-6x)^2}}} + C; C \in \mathbb{R}$

c)  $\int \sin\left(\frac{3x-5}{2}\right) dx = \begin{cases} \frac{3x-5}{2}=t \\ \frac{1}{2} \cdot 3 dx=dt \end{cases} = \frac{2}{3} \int \sin t dt = -\frac{2}{3} \cos\left(\frac{3x-5}{2}\right) + C; C \in \mathbb{R}$

d)  $\int \frac{1}{\sin^2\left(\frac{x-2}{3}\right)} dx = \begin{cases} \frac{x-2}{3}=t \\ \frac{1}{3} dx=dt \end{cases} = 3 \int \frac{1}{\sin^2 t} dt = -3 \cot g\left(\frac{x-2}{3}\right) + C; C \in \mathbb{R}$

e)  $\int \cot g(5x+9) dx = \begin{cases} 5x+9=t \\ 5dx=dt \end{cases} = \frac{1}{5} \int \frac{\cos t}{\sin t} dt = \frac{1}{5} \ln |\sin(5x+9)| + C; C \in \mathbb{R}$

(3.2.2) f)  $\int \frac{3}{\sqrt[3]{(5-2x)^3}} dx = \begin{cases} 5-2x=t \\ -2dx=dt \end{cases} = -\frac{1}{2} \cdot 3 \cdot \int \frac{1}{t^{\frac{3}{2}}} dt = -\frac{3}{2} \cdot \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + c = -\frac{3}{5} \cdot \sqrt{(5-2x)^5} + c; c \in \mathbb{R}$

g)  $\int (3-2x)^3 dx = \begin{cases} 3-2x=t \\ -2dx=dt \end{cases} = -\frac{1}{2} \int t^3 dt = -\frac{1}{2} \cdot \frac{t^4}{4} + c = -\frac{1}{8} \cdot (3-2x)^4 + c; c \in \mathbb{R}$

h)  $\int \frac{1}{(5+3x)^3} dx = \begin{cases} 5+3x=t \\ 3dx=dt \end{cases} = \frac{1}{3} \int \frac{1}{t^3} dt = \frac{1}{3} \cdot \frac{t^{-2}}{-2} + c = -\frac{1}{6} \cdot \frac{1}{(5+3x)^2} + c; c \in \mathbb{R}$

(3.2.3) a)  $\int \frac{2^x}{1+4^x} dx = \begin{cases} 2^x=t \\ 2^x \cdot \ln 2 dx = dt \end{cases} = \frac{1}{\ln 2} \cdot \int \frac{2^x \cdot \ln 2}{1+(2^x)^2} dx = \frac{1}{\ln 2} \cdot \int \frac{1}{1+t^2} dt = \frac{1}{\ln 2} \cdot \arctan 2^x + c; c \in \mathbb{R}$

b)  $\int \frac{x^2}{\sqrt{1-x^6}} dx = \begin{cases} x^3=t \\ 3x^2 dx = dt \end{cases} = \frac{1}{3} \int \frac{3x^2}{\sqrt{1-(x^3)^2}} dx = \frac{1}{3} \int \frac{1}{\sqrt{1-t^2}} dt = \frac{1}{3} \arcsin x^3 + c; c \in \mathbb{R}$

c)  $\int \frac{e^{\frac{1}{x}}}{x^2} dx = \begin{cases} x^{-1}=t \\ -x^{-2}dx=dt \end{cases} = - \int \frac{e^t}{1} dt = -e^{\frac{1}{x}} + c; c \in \mathbb{R}$

d)  $\int e^{\cos^2 x} \cdot \sin 2x dx = \begin{cases} \cos^2 x=t \\ 2\cos x \cdot (-\sin x) dx = dt \end{cases} = - \int e^t dt = -e^{\cos^2 x} + c; c \in \mathbb{R}$

plauti:  $\boxed{\sin 2x = 2 \cdot \sin x \cdot \cos x}$

(3.2.3) e)  $\int \frac{\sqrt{1+\ln x}}{x} dx = \begin{cases} 1+\ln x = t \\ \frac{1}{x} dx = dt \end{cases} = \int t^{\frac{1}{2}} dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3} \cdot \underline{\underline{(1+\ln x)^{\frac{3}{2}}}} + c; c \in \mathbb{R}$

f)  $\int \frac{1}{x \cdot \sqrt[3]{\ln 3x}} dx = \begin{cases} \ln 3x = t \\ \frac{1}{3x} \cdot 3dx = dt \end{cases} = \int \frac{1}{\sqrt[3]{t}} dt = \int t^{-\frac{1}{3}} dt = \frac{t^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{3}{2} \cdot \underline{\underline{(\ln 3x)^{\frac{2}{3}}}} + c; c \in \mathbb{R}$

g)  $\int \frac{\operatorname{tg} \sqrt{x}}{\sqrt{x}} dx = \begin{cases} x^{\frac{1}{2}} = t \\ \frac{1}{2} x^{-\frac{1}{2}} dx = dt \end{cases} = 2 \int \operatorname{tg} t dt = 2 \cdot \int \frac{\sin t}{\cos t} dt = -2 \cdot \underline{\underline{\ln |\cos t|}} + c; c \in \mathbb{R}$

h)  $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}} dx = \begin{cases} \sin x = t \\ \cos x dx = dt \end{cases} = \int \frac{1}{t^{\frac{2}{3}}} dt = \frac{t^{\frac{1}{3}}}{\frac{1}{3}} + c = \underline{\underline{\sqrt[3]{\sin x}}} + c; c \in \mathbb{R}$

(3.3.1) a)  $\int x \cdot \sin x dx = \begin{cases} f' = \sin x & g = x \\ f = -\cos x & g' = 1 \end{cases} = -x \cos x + \int \cos x dx = -x \cos x + \underline{\underline{\sin x}} + c; c \in \mathbb{R}$

Platir:  $(f \cdot g)' = f'g + fg' / -fg'$

$f \cdot g = (f \cdot g)' - f \cdot g' / \int$

$\int f \cdot g dx = \int (f \cdot g)' dx - \int f \cdot g' dx$

$\boxed{\int f'(x) \cdot g(x) dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x) dx} \rightarrow \text{Per Partes}$

(3.3.1)

$$\text{b) } \int x \cdot e^{2x} dx = \begin{vmatrix} f = e^{2x} & g = x \\ f = \frac{e^{2x}}{2} & g' = 1 \end{vmatrix} = x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} dx = \frac{x}{2} \cdot e^{2x} - \frac{1}{2} \int e^{2x} dx = \frac{x}{2} e^{2x} - \frac{1}{2} \cdot \frac{e^{2x}}{2} + C = \underline{\underline{\left( \frac{x}{2} - \frac{1}{4} \right) e^{2x} + C; C \in \mathbb{R}}} \quad \text{ceR}$$

$$\text{c) } \int (2x-5) \cdot \sin 3x dx = \begin{vmatrix} f = \sin 3x & g = 2x-5 \\ f = -\cos 3x & g' = 2 \end{vmatrix} = -\frac{\cos 3x}{3} \cdot (2x-5) - \int \frac{-\cos 3x}{3} \cdot 2 dx = -\frac{2x-5}{3} \cdot \cos 3x + \frac{2}{3} \int \cos 3x dx = \underline{\underline{-\frac{2x-5}{3} \cdot \cos 3x + \frac{2}{3} \cdot \frac{\sin 3x}{3} + C; C \in \mathbb{R}}}$$

$$\text{d) } \int (5x+2) \cdot 2^x dx = \begin{vmatrix} f = 2^x & g = 5x+2 \\ f = \frac{2^x}{\ln 2} & g' = 5 \end{vmatrix} = (5x+2) \cdot \frac{2^x}{\ln 2} - 5 \int \frac{2^x}{\ln 2} dx = (5x+2) \cdot \frac{2^x}{\ln 2} - \frac{5}{\ln 2} \cdot \frac{2^x}{\ln 2} + C = \underline{\underline{\frac{2^x}{\ln 2} \cdot \left( 5x+2 - \frac{5}{\ln 2} \right) + C; C \in \mathbb{R}}}$$

$$\text{e) } \int \frac{x}{\sin^2 x} dx = \begin{vmatrix} f = \frac{1}{\sin^2 x} & g = x \\ f = -\operatorname{cosec} x & g' = 1 \end{vmatrix} = -x \operatorname{cosec} x + \int \operatorname{cosec} x dx = -x \operatorname{cosec} x + \ln |\sin x| + C; C \in \mathbb{R}$$

$$\text{f) } \int (x^2 + 6x - 7) \cdot \cos x dx = \begin{vmatrix} f = \cos x & g = x^2 + 6x - 7 \\ f = \sin x & g' = 2x+6 \end{vmatrix} = (x^2 + 6x - 7) \sin x - \int (2x+6) \sin x dx = (x^2 + 6x - 7) \sin x + (2x+6) \cos x - 2 \sin x + \underline{\underline{+C; C \in \mathbb{R}}}$$

$$\int (2x+6) \sin x dx = \begin{vmatrix} f = \sin x & g = 2x+6 \\ f = -\cos x & g' = 2 \end{vmatrix} = -(2x+6) \cos x + \int 2 \cos x dx = \underline{\underline{-(2x+6) \cos x + 2 \sin x + C}}$$

(3.3.1) g)  $\int (4x-x^2) \cdot 5^x dx = \begin{cases} f = 5^x \\ f' = \frac{5^x}{\ln 5} \end{cases} \quad \begin{cases} g = 4x-x^2 \\ g' = 4-2x \end{cases} = \frac{5^x}{\ln 5} \cdot (4x-x^2) - \int \frac{5^x}{\ln 5} \cdot (4-2x) = \frac{5^x}{\ln 5} \cdot (4x-x^2) - \frac{1}{\ln 5} \cdot \int 5^x \cdot (4-2x) dx =$

$$= \frac{5^x}{\ln 5} \cdot (4x-x^2) - \frac{1}{\ln 5} \cdot \left[ \frac{5^x}{\ln 5} \cdot (4-2x) + \frac{2}{\ln^2 5} \cdot 5^x \right] = \underline{\underline{\frac{5^x}{\ln 5} \cdot \left( 4x-x^2 - \frac{4-2x}{\ln 5} - \frac{2}{\ln^2 5} \right)}} + C; C \in \mathbb{R}$$

$\int 5^x \cdot (4-2x) dx = \begin{cases} f = 5^x \\ f' = \frac{5^x}{\ln 5} \end{cases} \quad \begin{cases} g = 4-2x \\ g' = -2 \end{cases} = (4-2x) \cdot \frac{5^x}{\ln 5} + 2 \cdot \int \frac{5^x}{\ln 5} dx = \underline{\underline{\frac{5^x}{\ln 5} \cdot (4-2x) + \frac{2}{\ln 5} \cdot \frac{5^x}{\ln 5}}} + C$

h)  $\int (x^2+2x-3) \cdot e^{-x} dx = \begin{cases} f = e^{-x} \\ f' = -e^{-x} \end{cases} \quad \begin{cases} g = x^2+2x-3 \\ g' = 2x+2 \end{cases} = -e^{-x} \cdot (x^2+2x-3) + \int e^{-x} \cdot (2x+2) dx = -e^{-x} \cdot (x^2+2x-3+2x+4) =$

$$= \underline{\underline{-e^{-x} \cdot (x^2+4x+1)}} + C; C \in \mathbb{R}$$

$\int (2x+2) e^{-x} dx = \begin{cases} f = e^{-x} \\ f' = -e^{-x} \end{cases} \quad \begin{cases} g = 2x+2 \\ g' = 2 \end{cases} = -e^{-x} \cdot (2x+2) + 2 \cdot \int e^{-x} dx = -e^{-x} \cdot (2x+2+2) = \underline{\underline{-e^{-x} \cdot (2x+4)}} + C$

i)  $\int (3x+5) \cdot \cos \frac{x}{3} dx = \begin{cases} f = \cos \frac{x}{3} \\ f' = -\sin \frac{x}{3} \end{cases} \quad \begin{cases} g = 3x+5 \\ g' = 3 \end{cases} = 3 \cdot \sin \frac{x}{3} \cdot (3x+5) - 9 \cdot \int \sin \frac{x}{3} dx = 3 \cdot \sin \frac{x}{3} \cdot (3x+5) + 9 \cdot \cos \frac{x}{3} \cdot 3 + C =$

$$= \underline{\underline{3 \cdot (3x+5) \cdot \sin \frac{x}{3} + 27 \cdot \cos \frac{x}{3}}} + C; C \in \mathbb{R}$$

derivácia  $(\cos \frac{x}{3})' = -\sin \frac{x}{3} \cdot \frac{1}{3}$

integral a derivácia sú navzájom inverzné (opäťné) funkcie,

pričo integral  $\int \cos \frac{x}{3} dx$  nebude  $\sin \frac{x}{3} \cdot \frac{1}{3}$ , ale  $\sin \frac{x}{3} \cdot 3$ , tak aby platilo:  $(3 \cdot \sin \frac{x}{3})' = \cos \frac{x}{3}$

(3.3.1) j)  $\int (2x-7) \cdot \ln^2 x \, dx = \begin{vmatrix} f = \ln^2 x & g = 2x-7 \\ f = \ln x - x & g = 2 \end{vmatrix} = \textcircled{*}$

$$\int \ln^2 x \, dx = \int \left( \frac{\sin^2 x}{\cos x} \right)^2 \, dx = \int \frac{\sin^2 x}{\cos^2 x} \, dx = \int \left( \frac{1 - \cos^2 x}{\cos^2 x} - 1 \right) \, dx = \underline{\ln x - x + c_1}$$

$$\begin{aligned} \textcircled{*} &= (\ln x - x) \cdot (2x-7) - 2 \cdot \int (\ln x - x) \, dx = (\ln x - x)(2x-7) - 2 \cdot \left[ \frac{1}{2} \ln |\cos x| - x \right] + c = \\ &= \underline{(\ln x - x)(2x-7) + 2 \cdot \ln |\cos x| + x^2 + c; c \in \mathbb{R}} \end{aligned}$$

(3.3.2) a)  $\int x^3 \cdot \ln x \, dx = \begin{vmatrix} f = x^3 & g = \ln x \\ f = \frac{x^4}{4} & g = \frac{1}{x} \end{vmatrix} = \frac{x^4}{4} \cdot \ln x - \frac{1}{4} \cdot \int \frac{x^4}{x} \, dx = \frac{x^4}{4} \cdot \ln x - \frac{1}{4} \int x^3 \, dx = \frac{x^4}{4} \cdot \ln x - \frac{1}{4} \cdot \frac{x^4}{4} + c = \\ &= \underline{\frac{x^4}{4} \cdot \left( \ln x - \frac{1}{4} \right) + c; c \in \mathbb{R}}$

$$\begin{aligned} b) \int \arccos x \, dx &= \int (1 \cdot \arccos x) \, dx = \begin{vmatrix} f = 1 & g = \arccos x \\ f = x & g = -\frac{1}{1+x^2} \end{vmatrix} = x \cdot \arccos x + \int \frac{x}{1+x^2} \, dx = \\ &= x \cdot \arccos x + \frac{1}{2} \int \frac{2x}{1+x^2} \, dx = \\ &= x \cdot \arccos x + \frac{1}{2} \cdot \ln |1+x^2| + c; c \in \mathbb{R} \end{aligned}$$

(3.3.2)

$$c) \int \arccos x \, dx = \begin{cases} f' = 1 & g = \arccos x \\ f = x & g' = -\frac{1}{\sqrt{1-x^2}} \end{cases} = x \cdot \arccos x + \int \frac{x}{\sqrt{1-x^2}} \, dx = x \cdot \arccos x - \sqrt{1-x^2} + c; c \in \mathbb{R}$$

$$\int \frac{x}{\sqrt{1-x^2}} \, dx = \begin{cases} \text{subst.} \\ 1-x^2 = t \\ \rightarrow x \, dx = dt \end{cases} = -\frac{1}{2} \int \frac{1}{\sqrt{t}} \, dt = -\frac{1}{2} \int t^{-\frac{1}{2}} \, dt = -\frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = -\sqrt{1-x^2} + c$$

$$d) \int \operatorname{arccotg} 2x \, dx = \begin{cases} f' = 1 & g = \operatorname{arccotg} 2x \\ f = x & g' = -\frac{1}{1+(2x)^2} \cdot 2 \end{cases} = x \cdot \operatorname{arccotg} 2x + 2 \int x \cdot \frac{1}{1+4x^2} \, dx = x \cdot \operatorname{arccotg} 2x + \frac{1}{4} \ln |1+4x^2| + c$$

$$\int \frac{x}{1+4x^2} \, dx = \frac{1}{8} \int \frac{8x}{1+4x^2} \, dx = \frac{1}{8} \cdot \ln |1+4x^2| + c$$

$$e) \int x \cdot \ln^2 x \, dx = \begin{cases} f' = x & g = \ln^2 x \\ f = \frac{x^2}{2} & g' = 2 \cdot \ln x \cdot \frac{1}{x} \end{cases} = \frac{x^2}{2} \cdot \ln^2 x - \int \frac{x^2}{2} \cdot \frac{2}{x} \cdot \ln x \, dx = \frac{x^2}{2} \ln^2 x - \int x \cdot \ln x \, dx = \textcircled{*}$$

$$\int x \cdot \ln x \, dx = \begin{cases} f' = x & g = \ln x \\ f = \frac{x^2}{2} & g' = \frac{1}{x} \end{cases} = \frac{x^2}{2} \cdot \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} \, dx = \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx = \frac{x^2}{2} \ln x - \frac{1}{2} \cdot \frac{x^2}{2} + c$$

$$\textcircled{*} = \frac{x^2}{2} \ln^2 x - \left( \frac{x^2}{2} \ln x - \frac{1}{4} x^2 \right) = \frac{x^2}{2} \ln^2 x - \frac{x^2}{2} \ln x + \frac{1}{4} x^2 = \frac{x^2}{2} \cdot \left( \ln^2 x - \ln x + \frac{1}{2} \right) + c; c \in \mathbb{R}$$

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(3.3.2) f)  $\int \frac{\ln x}{x^3} dx = \begin{cases} f = x^{-3} \\ f = \frac{x}{x^{-2}} \end{cases} \quad \begin{cases} g = \ln x \\ g = \frac{1}{x} \end{cases} = \frac{-\ln x}{2x^2} - \int \frac{-1}{2x^2} \cdot \frac{1}{x} dx = \frac{-\ln x}{2x^2} + \frac{1}{2} \cdot \int x^{-3} dx =$

$$= -\frac{\ln x}{2x^2} + \frac{1}{2} \cdot \frac{x^{-2}}{-2} + C = -\frac{\ln x}{2x^2} - \frac{1}{4x^2} + C = -\frac{1}{2x^2} \cdot \left( \ln x + \frac{1}{2} \right) + C; C \in \mathbb{R}$$

h)  $\int x^2 \cdot \operatorname{arctg} x dx = \begin{cases} f = x^2 \\ f = \frac{x^3}{3} \end{cases} \quad \begin{cases} g = \operatorname{arctg} x \\ g = \frac{1}{1+x^2} \end{cases} = \frac{x^3}{3} \cdot \operatorname{arctg} x - \frac{1}{3} \int \frac{x^3}{1+x^2} dx = \textcircled{*}$

$$\int \frac{x^3}{1+x^2} dx = \frac{1}{2} \int \frac{x^2 \cdot 2x}{1+x^2} dx = \begin{cases} 1+x^2 = t \\ 2x dx = dt \\ x^2 = t-1 \end{cases} = \frac{1}{2} \int \frac{t-1}{t} dt = \frac{1}{2} \cdot \left( \int 1 dt - \int \frac{1}{t} dt \right) =$$

$$= \frac{1}{2} \cdot \left( t - \ln|t| \right) + C = \underline{\frac{1}{2} \cdot (1+x^2) - \frac{1}{2} \ln|1+x^2| + C}; C \in \mathbb{R}$$

$\textcircled{*} = \frac{x^3}{3} \operatorname{arctg} x - \frac{1}{3} \cdot \left[ \frac{1}{2} (1+x^2) - \frac{1}{2} \ln|1+x^2| \right] = \underline{\frac{x^3}{3} \operatorname{arctg} x - \frac{1}{6} \cdot (1+x^2) + \frac{1}{6} \ln|1+x^2| + C}; C \in \mathbb{R}$

(3.4.5)

a)  $f_1: y = -x^2 + 2x + 8$

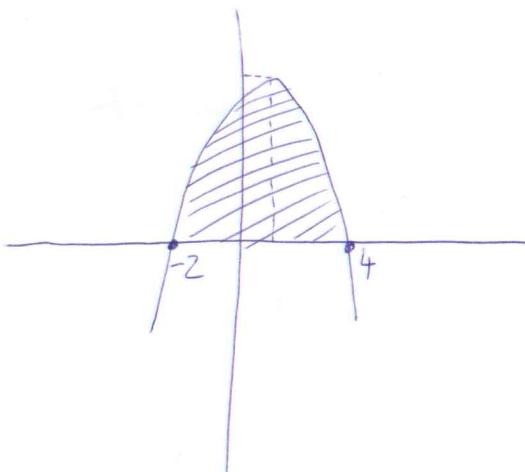
$f_2: y = 0$

Položíme  $f_1 = f_2$  a zistíme hraničné body, teda  $\underline{a}$  a  $\underline{b}$ :

$$-x^2 + 2x + 8 = 0$$

$$D = 4 + 4 \cdot 8 = 36$$

$$x_{1,2} = \frac{-2 \pm \sqrt{36}}{-2} = \underline{-2}, \underline{4}$$



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Platí: Nech  $a < b$ ;  $a, b \in \mathbb{R}$ . Nech  $g(x) \leq f(x)$  na intervalu  $\langle a; b \rangle$ .

Potom:

$$S = \int_a^b [f(x) - g(x)] dx$$

je obsah útvary ohraniceného zhorou funkciou  $f(x)$ , zolou funkciou  $g(x)$ , zľava bodom  $\underline{a}$  a zprava bodom  $\underline{b}$ .

Platí:

Newton-Leibnizov vzorec:

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a), \text{ kde } F'(x) = f(x)$$

$$\begin{aligned}
 S &= \int_{-2}^4 [(-x^2 + 2x + 8) - 0] dx = \int_{-2}^4 (-x^2 + 2x + 8) dx = \left[ -\frac{x^3}{3} + 2 \cdot \frac{x^2}{2} + 8x \right]_2^4 = \\
 &= \left( -\frac{4^3}{3} + 4^2 + 8 \cdot 4 \right) - \left( -\frac{(-2)^3}{3} + (-2)^2 + 8 \cdot (-2) \right) = \left( -\frac{64}{3} + 16 + 32 \right) - \left( \frac{8}{3} + 4 - 16 \right) = \\
 &= -\frac{64}{3} - \frac{8}{3} + 48 + 12 = \frac{-72}{3} + 60 = -24 + 60 = \underline{\underline{36}}
 \end{aligned}$$

(304.5)

$$b) f_1: y = 16 - x^2$$

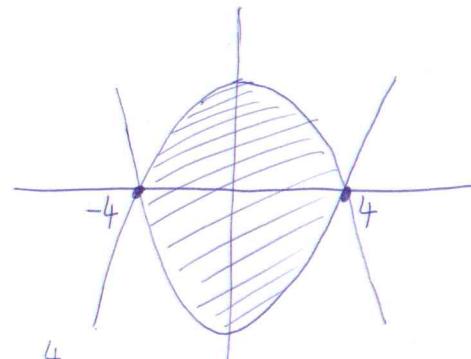
$$f_2: y = x^2 - 16$$

$$\begin{aligned} f_1 &= f_2 \\ 16 - x^2 &= x^2 - 16 \end{aligned}$$

$$2x^2 = 32$$

$$x^2 = 16$$

$$x_{1,2} = \pm 4$$



$$S = \int_{-4}^4 [(16 - x^2) - (x^2 - 16)] dx = \int_{-4}^4 2 \cdot (16 - x^2) dx = 2 \cdot \int_{-4}^4 (16 - x^2) dx =$$

$$= 2 \cdot \left[ 16x - \frac{x^3}{3} \right]_{-4}^4 = 2 \cdot \left( 16 \cdot 4 - \frac{4^3}{3} - \left( 16 \cdot (-4) - \frac{(-4)^3}{3} \right) \right) =$$

$$= 2 \cdot \left( 64 - \frac{64}{3} + 64 - \frac{64}{3} \right) = 4 \cdot \left( 64 - \frac{64}{3} \right) = 4 \cdot \frac{192 - 64}{3} = \underline{\underline{\frac{512}{3}}} = 170\bar{6}$$

$$c) f_1: y = x^2 - 4x + 6$$

$$f_2: y = -2x^2 + 8x - 3$$

$$f_1 = f_2$$

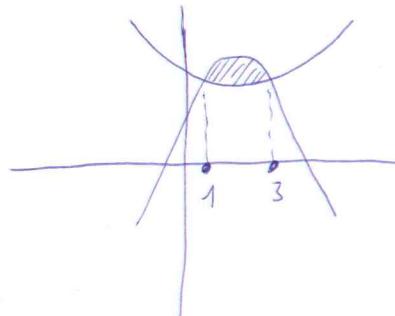
$$x^2 - 4x + 6 = -2x^2 + 8x - 3$$

$$3x^2 - 12x + 9 = 0 \quad | :3$$

$$x^2 - 4x + 3 = 0$$

$$D = 16 - 4 \cdot 3 = 4$$

$$x_{1,2} = \frac{4 \pm 2}{2} = \begin{cases} 1 \\ 3 \end{cases}$$



$$S = \int_{1}^3 (x^2 - 4x + 6 + 2x^2 - 8x + 3) dx =$$

$$= \int_{1}^3 (3x^2 - 12x + 9) dx =$$

$$= \left[ 3 \cdot \frac{x^3}{3} - 12 \cdot \frac{x^2}{2} + 9x \right]_1^3 = \left[ x^3 - 6x^2 + 9x \right]_1^3 =$$

$$= 27 - 54 + 27 - 1 + 6 - 9 = \underline{-4} \Rightarrow |-4| = \underline{\underline{4}}$$

→ Aknám výjde záporný výsledok, obsah útvary je absolútne hodnota z tohto čísla!

(108)

(3.4.5)

$$d) f_1: y = x^2 + 6x + 8$$

$$f_1 = f_2$$

$$f_2: y = -x^2 - 10x - 16$$

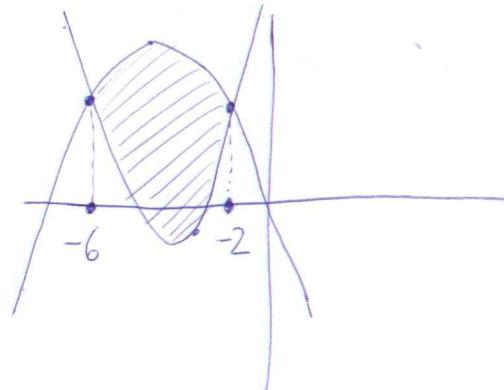
$$x^2 + 6x + 8 = -x^2 - 10x - 16$$

$$2x^2 + 16x + 24 = 0 \quad | :2$$

$$x^2 + 8x + 12 = 0$$

$$D = 64 - 4 \cdot 12 = 16$$

$$x_{1,2} = \frac{-8 \pm 4}{2} = \begin{cases} -6 \\ -2 \end{cases}$$



(109)

$$\begin{aligned} S &= \int_{-6}^{-2} (x^2 + 6x + 8 + x^2 + 10x + 16) dx = \int_{-6}^{-2} (2x^2 + 16x + 24) dx = \\ &= \left[ 2 \cdot \frac{x^3}{3} + 16 \cdot \frac{x^2}{2} + 24x \right]_{-6}^{-2} = \\ &= \left[ 2 \cdot \frac{(-2)^3}{3} + 16 \cdot \frac{(-2)^2}{2} + 24 \cdot (-2) - 2 \cdot \frac{(-6)^3}{3} - 8 \cdot (-6)^2 + 24 \cdot (-6) \right] = \\ &= -\frac{2}{3} \cdot 8 + 32 - 48 + \frac{2}{3} \cdot 216 - 288 + 144 = -\frac{64}{3} \Rightarrow S = \underline{\underline{\frac{64}{3} - 21,3}} \end{aligned}$$

$$e) f_1: y = -9 - x^2$$

$$f_2: y = -5x - 9$$

$$f_1 = f_2$$

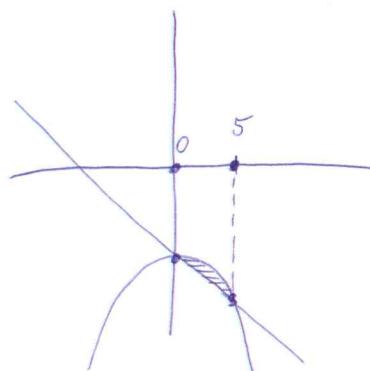
$$-9 - x^2 = -5x - 9$$

$$-x^2 + 5x = 0 \quad | \cdot (-1)$$

$$x(x-5) = 0$$

$$x_1 = 0$$

$$\boxed{x_2 = 5}$$



$$\begin{aligned} S &= \int_0^5 (-9 - x^2 + 5x + 9) dx = \int_0^5 (5x - x^2) dx = \\ &= \left[ 5 \cdot \frac{x^2}{2} - \frac{x^3}{3} \right]_0^5 = \left( 5 \cdot \frac{5^2}{2} - \frac{5^3}{3} \right) = \end{aligned}$$

$$\frac{5^3}{2} - \frac{5^3}{3} = \frac{3 \cdot 5^3 - 2 \cdot 5^3}{6} = \frac{125}{6} = \underline{\underline{20,8}}$$

30405

$$f) f_1: y = x^2$$

$$f_1 = f_2$$

$$f_{g_1} = f_3$$

$$f_2 = f_3$$

$$f_2: y = 2x^2$$

$$x^2 = 2x^2$$

$$x^2 = 1$$

$$2x^2 = 1$$

$$f_3: y=1$$

$$3x^2 = 0$$

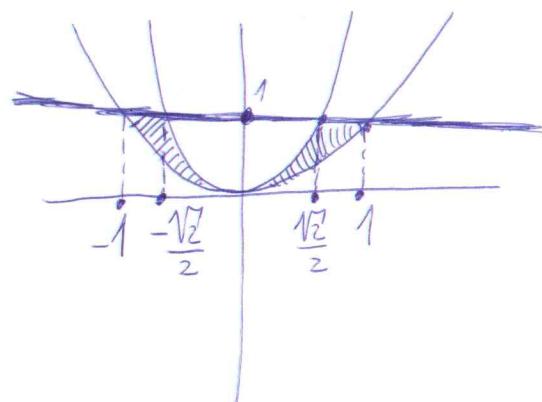
$$x = \pm 1$$

$$x^2 = \frac{1}{2} \quad | \sqrt{\cdot}$$

$$\boxed{x = 0}$$

$$x = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}$$

$$x = \pm \frac{\sqrt{2}}{2}$$



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(~~all species of the family Serruridae probably over 50, still listed~~)

$\rightarrow$  nech  $S_1$  je obsah útvaru ohrazeného závařem  $\boxed{-1}$ , zprava  $\boxed{1}$ , zhora  $\boxed{y=1}$ , zdola  $\boxed{y=x}$   
 nech  $S_2$   $\rule{1cm}{0.4pt}$  //  $\rule{1cm}{0.4pt}$   $\boxed{-\frac{\sqrt{2}}{2}}$ , sprava  $\boxed{\frac{\sqrt{2}}{2}}$ , zhora  $\boxed{y=1}$ , zdola  $\boxed{y=2x^2}$

$$\text{Potom } S = S_1 - S_2 .$$

$$S_1 = \int_{-1}^1 (1-x^2) dx = \left[ x - \frac{x^3}{3} \right]_{-1}^1 = 1 - \frac{1}{3} - \left( -1 + \frac{1}{3} \right) = 2 - \frac{2}{3} = \underline{\frac{4}{3}}$$

$$S_2 = \int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \left(1 - 2x^2\right) dx = \left[x - 2 \cdot \frac{x^3}{3}\right]_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2} - \frac{2}{3} \cdot \left(\frac{\sqrt{2}}{2}\right)^3 - \left(-\frac{\sqrt{2}}{2} - \frac{2}{3} \cdot \left(-\frac{\sqrt{2}}{2}\right)^3\right) = 2 \cdot \frac{\sqrt{2}}{2} - 2 \cdot \frac{2}{3} \cdot \left(\frac{\sqrt{2}}{2}\right)^3 = \frac{\sqrt{2}}{2} - \frac{4}{3} \cdot \frac{(\sqrt{2})^3}{8} = \frac{\sqrt{2}}{2} - \frac{4}{3} \cdot \frac{2 \cdot \sqrt{2}}{8} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{3} = \frac{3\sqrt{2} - \sqrt{2}}{3} = \frac{2\sqrt{2}}{3}$$

$$S = S_1 - S_2 = \frac{4}{3} - \frac{2\sqrt{2}}{3} = \frac{2 \cdot (2 - \sqrt{2})}{3} = \underline{\underline{\frac{2 \cdot (2 - \sqrt{2})}{3}}}$$

(30.4.5)

$$g) f_1: y = 2x^3$$

$$f_1 = f_2 \\ 2x^3 = \frac{x}{2}$$

$$f_2: y = \frac{x}{2}$$

$$2x^3 - \frac{x}{2} = 0 \quad | :2$$

$$x(x^2 - \frac{1}{4}) = 0 \Rightarrow x=0 \vee x = \pm \frac{1}{2}$$

$S = \int_{-1/2}^{1/2} (2x^3 - \frac{1}{2}x) dx = \left[ 2 \cdot \frac{x^4}{4} - \frac{1}{2} \cdot \frac{x^2}{2} \right]_{-1/2}^{1/2} = \left[ \frac{1}{2}x^4 - \frac{1}{4}x^2 \right]_{-1/2}^{1/2} = \frac{1}{2} \cdot \left(\frac{1}{2}\right)^4 - \frac{1}{4} \cdot \left(\frac{1}{2}\right)^2 = -\frac{1}{32} \Rightarrow S = 2 \cdot |S'| = 2 \cdot \frac{1}{32} = \underline{\underline{\frac{1}{16}}}$

(111)

$$h) f_1: y = x$$

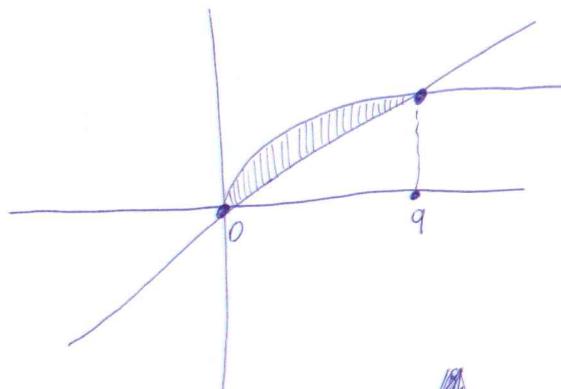
$$f_1 = f_2$$

$$f_2: y = 3\sqrt[3]{x}$$

$$x = 3\sqrt[3]{x} \quad |^2$$

$$x^2 - 9x = 0$$

$$x(x-9) = 0 \Rightarrow x=0 \vee x=9$$



$$S = \int_0^9 (x - 3\sqrt[3]{x}) dx = \int_0^9 (x - 3 \cdot x^{\frac{1}{3}}) dx = \left[ \frac{x^2}{2} - 3 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^9 = \left[ \frac{x^2}{2} - 2 \cdot \sqrt[3]{x^3} \right]_0^9 = \frac{81}{2} - 2 \cdot \sqrt[3]{9^3} = \frac{81}{2} - 2 \cdot 9 \cdot \sqrt[3]{9} = \frac{81}{2} - 54 = \frac{81-108}{2} = \underline{\underline{-\frac{27}{2}}}$$

$$\Rightarrow S = \underline{\underline{\frac{27}{2}}} = 13,5$$